MODELLING AND CONTROL OF A TETHERED KITE IN DYNAMIC FLIGHT

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The need in reducing the CO₂ emissions and the increasing oil prices affect all transportation industries and especially the maritime industry. This induces to redesign propulsion systems of ships to spare energy. In this context, taking advantage of wind energy by using tethered airfoils, or kites, as an alternative propulsion source can be an effective solution. The Beyond the Sea project, led by Yves Parlier, aims to provide ships an alternative green energy source.

The need in reducing the CO₂ emissions and the increasing oil prices affect all transportation industries and especially the maritime industry. This induces to redesign propulsion systems of ships to spare energy. In this context, taking advantage of wind energy by using tethered airfoils, or kites, as an alternative propulsion source can be an effective solution [17]. It involves an AWE (Airborne Wind Energy) concept, operating at high altitudes, where the wind is stronger and steadier. AWE concepts can be used to convert traction power of tethered airfoils into electricity effectively [13], [6] or into ship propulsion.

In order to offer a wind propulsion system for ships, a towing kite following a dynamic trajectory is the most interesting candidate for such a purpose [15], [22]. Indeed, contrarily to a static flight, dynamic motion of a tethered wing in “8” shape pattern can provide sufficient energy through traction for towing ships. The advantages are an endless source of energy from the wind, as well as a compact, light-weight system that can be used or put away easily taking very little space. This concept can be very beneficial for any ship for both economical and environmental efficiency. The wing itself becomes the most crucial part of the system. With increasing size, traditional empirical approaches are too slow and the costs of prototypes too expensive. As such, obtaining a satisfying model to understand its dynamical behaviour is necessary.

In this paper, the focus is on modelling a tethered wing and building a realistic simulator which can be used to design a robust controller, so that the kite follows a desired trajectory. Section 2 presents the theoretical model of the wing, also called kite. Section 3 deals with experimental test performed.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>A_k</td>
<td>Kite surface</td>
<td>m²</td>
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<tr>
<td>C_D</td>
<td>Kite drag coefficient</td>
<td>-</td>
</tr>
<tr>
<td>C_L</td>
<td>Kite lift coefficient</td>
<td>-</td>
</tr>
<tr>
<td>C_t</td>
<td>Kite lateral force coefficient</td>
<td>-</td>
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<tr>
<td>d</td>
<td>Distance from kite to trajectory</td>
<td>m</td>
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<tr>
<td>d₀</td>
<td>Pivot distance for kite orientation</td>
<td>m</td>
</tr>
<tr>
<td>F</td>
<td>Force vector</td>
<td>N</td>
</tr>
<tr>
<td>F_a</td>
<td>Aerodynamic resultant force vector</td>
<td>N</td>
</tr>
<tr>
<td>F_g</td>
<td>Gravitational force vector</td>
<td>N</td>
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<tr>
<td>g_k</td>
<td>Proportional gain of turn rate law</td>
<td>-</td>
</tr>
<tr>
<td>m</td>
<td>Kite mass</td>
<td>kg</td>
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<tr>
<td>M</td>
<td>Gravitational gain of turn rate law</td>
<td>-</td>
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<tr>
<td>R_WT</td>
<td>True wind reference frame</td>
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<td>R_WR</td>
<td>Relative wind reference frame</td>
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<tr>
<td>R_k0</td>
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<tr>
<td>R_b</td>
<td>Body reference frame</td>
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<tr>
<td>R_t</td>
<td>Trajectory reference frame</td>
<td>-</td>
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<tr>
<td>r</td>
<td>line’s length</td>
<td>m</td>
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<tr>
<td>V_a</td>
<td>Kite apparent wind velocity vector</td>
<td>m.s⁻¹</td>
</tr>
<tr>
<td>V_k</td>
<td>Kite velocity vector</td>
<td>m.s⁻¹</td>
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<tr>
<td>V_cmd</td>
<td>Desired kite velocity vector</td>
<td>m.s⁻¹</td>
</tr>
<tr>
<td>V_WR</td>
<td>True wind velocity vector</td>
<td>m.s⁻¹</td>
</tr>
<tr>
<td>v_a</td>
<td>Apparent wind velocity projected on kite x-axis</td>
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</tr>
<tr>
<td>x_k</td>
<td>Kite position vector</td>
<td>-</td>
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<tr>
<td>α</td>
<td>Angle of attack</td>
<td>rad</td>
</tr>
<tr>
<td>θ</td>
<td>Elevation angle in R_WR</td>
<td>rad</td>
</tr>
<tr>
<td>φ</td>
<td>Azimuth angle in R_WR</td>
<td>rad</td>
</tr>
<tr>
<td>ρ</td>
<td>Density of air</td>
<td>kg.m⁻³</td>
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<tr>
<td>ψ</td>
<td>Body yaw angle</td>
<td>rad</td>
</tr>
<tr>
<td>χ</td>
<td>Body velocity angle</td>
<td>rad</td>
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<tr>
<td>χ_c</td>
<td>Trajectory orientation</td>
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<tr>
<td>χ_cmd</td>
<td>Desired body velocity angle</td>
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by *Beyond the Sea* with a kite and sensor unit enabling the parameterization of the model. Section 4 provides a strategy for the kite to follow designated “8” shape pattern.

## 2 KITE MODELLING

### 2.1 SYSTEM DESCRIPTION

#### 2.1.1 Context

This section aims at delivering a suitable model for controlling a kite around a defined trajectory. Several models have been proposed in the literature. Each tries to replicate the dynamic motion of a kite, from low complexity models to very detailed ones. A zero mass model such as those from [16], [3] and [15], is not entirely suited for robust control because of its low complexity and accuracy, whereas the most advanced models taking into account the mechanics of deformable solids or multi-body systems [1] [2], [5], [4] are too resource consuming for efficient real-time control. In order to keep a sufficiently realistic model which can also be easily implemented in a controller, a point-mass model was chosen, as it offers sufficient accuracy while allowing a fast computing time. Such models have already been proposed in [8], [20], [11] or [7]. However, this paper proposes to include some additional aspects of the kite behaviour in dynamic flight in the point-mass model.

#### 2.1.2 Wing, tethers and hydraulic drives

As mentioned in the introduction, the main part of the system is the wing. The flexibility of the wing is not considered here, as the kite is assimilated to a rigid body, while the entire mass is concentrated in a point at the centre of the wing. The intended use of the system is the propulsion of a ship through the generated traction force. Conveying this energy implies a tethered wing arranged as follow: three lines are bound to the airfoil towards a fixed point of a ship deck. Two lines placed on the trailing edge of the kite are meant to be steering lines, and permit motion and attitude control. The last line, distributed on the leading edge of the kite, is the traction line and holds the most of the generated traction force.

The steering lines are considered constant in length and non-deformable. Their effects on the wing dynamical behaviour have been neglected and remain out the scope of this paper. The focus is on a kite wing dynamic modelling and control. In the following, the ship relative wind will be treated as the true wind from the kite point of view. On the anchor point, two hydraulic drives can simultaneously reel in or out the steering lines to control the wing. An asymmetric length difference of the back steering lines affects the yaw angle of the kite, meanwhile, slacking and hauling is done through adjusting the back lines symmetrically and will then influence the angle of attack. The traction line is left untouched.

### 2.2 POINT MASS MODEL

#### 2.2.1 Coordinate references

The model described in this paper is based on several coordinate systems. As shown in Figure 1, the main coordinate reference \( R_{WR} = (O, x_{WR}, y_{WR}, z_{WR}) \) is attached to the anchor point of the ship, labelled O. Throughout this paper, vectors will be denoted by bold letters. The apparent wind relative to the ship is used as the real wind seen by the kite. Hence, the \( x_{WR} \) axis is oriented according to the ship relative wind. The \( z_{WR} \) axis points downwards with respect to gravity. The kite position is parameterized using classical polar coordinates denoted \( (\theta, \phi, t)_{WR} \), where \( \theta \) is the elevation of the kite and \( \phi \) is the azimuth relative to time. \( r \) is the length of the steering lines and remains constant throughout the entire model. This reference frame defines a quarter sphere, which depicts the wind window of the kite. As suggested previously, the effects of the lines are neglected in the scope of this paper.

![Figure 1: Definition of coordinate references](image)

With respect to the basis vectors \( (x_{WR}, y_{WR}, z_{WR}) \), the kite position can be deduced using the elevation and azimuth as follow:

\[
\mathbf{x}_K = r \begin{bmatrix} \cos(\phi) \cos(\theta) \\ \sin(\phi) \cos(\theta) \\ -\sin(\theta) \end{bmatrix} / R_{WR}.
\] (1)

The second coordinate system \( R_{K0} = (K, x_{K0}, y_{K0}, z_{K0}) \) is centred at the position of the kite, labelled K. Its unit vector \( z_{K0} \) always points towards the reference point, i.e. the anchor point O of the ship. Its \( x_{K0} \) unit vector points downwards and in opposite direction of the zenith. Thereafter, the body reference frame system \( R_{K} = (K, x_{K}, y_{K}, z_{K}) \) is computed to account for the kite attitude and is also positioned at the kite location. As a result, a rotation around the \( z_{K} \) axis represents a change in the kite yaw angle \( \psi \), between the longitudinal \( x_{K} \) axis and the kite current geodesic, represented by \( -x_{K0} \), as explained in [16] [11]. Equivalently, a rotation around the \( y_{K} \) axis models the slacking and hauling of the kite, due to a symmetrical steering input by the two hydraulic drives.
2.2.2 Motion equations

By applying the second Newton’s law of motion to the kite in the reference frame $R_{k0}$, the elevation and azimuth accelerations can be computed by the following equations:

$$\ddot{\theta}/R_{k0} = -\frac{F_x}{m} - \sin(\theta) \cos(\theta) \dot{\phi}^2$$ \hspace{1cm} (2)

$$\ddot{\phi}/R_{k0} = \frac{F_y}{m \sin(\theta)} - 2 \tan(\theta) \dot{\theta} \dot{\phi}$$ \hspace{1cm} (3)

where $m$ is the kite mass and $r$ is the tethered length. Here, $\mathbf{F}$ denotes the sum of all forces applied at the kite centre of mass, with respect to $R_{k0}$. This global force is composed of all aerodynamic forces labelled $\mathbf{F}_a$, as well as the gravitational force $\mathbf{F}_g$.

$$\mathbf{F} = \mathbf{F}_a + \mathbf{F}_g.$$ \hspace{1cm} (4)

2.2.3 Aerodynamic forces

The aerodynamic forces $\mathbf{F}_a$ depend mainly on the apparent wind velocity vector $\mathbf{V}_a$ seen by the kite. As usual in aerodynamics, the apparent wind is calculated by the expression $\mathbf{V}_a = \mathbf{V}_{WR} - \mathbf{V}_k$. Therefore, it can be convenient to compute an aerodynamic reference frame, related to apparent wind velocity, which will give the direction of the drag $\mathbf{x}_\text{drag}$ and lift $\mathbf{x}_\text{lift}$. The drag force has the same direction as the apparent wind seen by the kite, as show in Figure 2. The lift is perpendicular to the drag and to the transverse axis $\mathbf{y}_b$ of the kite. Thus, if the reference frame $R_a = (K, \mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a)$ is defined with its unit vector $\mathbf{x}_a$ parallel and opposite to the apparent wind velocity vector, the direction of drag becomes known. Therefore, the computation of the lift direction is immediate:

$$\begin{cases}
\mathbf{x}_\text{lift} = -\mathbf{x}_a \\
\mathbf{x}_\text{drag} = \mathbf{x}_\text{lift} \times \mathbf{y}_b
\end{cases}$$ \hspace{1cm} (5)

Hence, the aerodynamic forces are given by:

$$\mathbf{F}_a = \frac{1}{2} \rho C_L A_k |\mathbf{V}_a|^2 \mathbf{x}_\text{lift} + \frac{1}{2} \rho C_D A_k |\mathbf{V}_a|^2 \mathbf{x}_\text{drag},$$ \hspace{1cm} (6)

where $\rho$ is the air density, and $A_k$ is the effective surface of the kite. The aerodynamic coefficients $C_L$ and $C_D$ are detailed in section 2.2.3.

It is a common assumption that the kite heading $\mathbf{x}_b$ always tries to stay in front of its apparent wind, as to be parallel with its perceived airflow. Thus, in this paper, we add a third aerodynamic force meant to align the kite with its apparent wind velocity vector. In order to represent that behaviour, a transverse force is added to the other aerodynamic forces, perpendicular to the drag and lift forces completing a right-hand reference frame. Thus, the direction of this force $\mathbf{x}_t$ is the opposite of the kite transverse axis $\mathbf{y}_b$, such as $\mathbf{x}_t = -\mathbf{y}_b$. The $\mathbf{y}_b$ axis try to remain perpendicular to the apparent wind, which inevitably leads to the alignment of the kite heading with its airflow. Being an aerodynamic force, its expression is similar to the drag or lift force and possesses a proper coefficient $C_t$. The transverse force $\mathbf{F}_t$ is given by:

$$\mathbf{F}_t = \frac{1}{2} \rho C_t A_k |\mathbf{V}_a|^2 \mathbf{x}_t.$$ \hspace{1cm} (7)

Thus, the three aerodynamic forces from (6) and (7) are added to form the global aerodynamic force $\mathbf{F}_a$. Figure 2 represents the drag and lift forces direction that act upon the kite centre of gravity.

2.2.4 Aerodynamic coefficients

Some models consider the aerodynamic coefficients as constant. Nonetheless, it is a known fact that the drag aerodynamic coefficients $C_L$ and $C_D$, and additionally the lateral force aerodynamic coefficient $C_t$, are related to the angle of attack $\alpha$. Figure 2 pictures the definition of the angle of attack as the angle between the apparent wind and the kite body. This angle can be decomposed linearly by separating $\alpha$ with the incidence of the wind relative to the tangent plane on the sphere $(x_{k0}, y_{k0})$, and the incidence between the tangent plane and the body plane $(x_b, y_b)$. The angle is defined by:

$$\alpha = \alpha_0 + \alpha_i,$$ \hspace{1cm} (8)

where $\alpha_i$ represents the incidence, or slacking and hauling, of the kite, by performing a symmetrical steering input $\epsilon$. This leads to a rotation of the kite around its transverse axis $\mathbf{y}_b$.

In the literature, drag and lift profile already exists for plane wings. Yet, profiles for kite wing are scarce and generally specific to a particular kite.

2.2.5 Turn rate law

The heading angle, or yaw angle, noted $\psi$ defines the relative direction $\mathbf{x}_{k0}$ where the kite is pointing. It is also important to note that the heading angle $\psi$ can differ from the actual flight.
direction, which is represented by its velocity vector. As such
the kite velocity angle \( \chi \) will be slightly different than \( \psi \),
especially when turning. In straight flight however, the two
angles are almost identical. The difference between the two is
called the drift angle and can be defined by \( \beta = \chi - \psi \). This
distinction is important when considering a path-following
kite, as in Section 4.

As opposed to a symmetrical steering input, the influence
of a differential steering, or asymmetrical, for the kite is
essentially related to its angular velocity. Indeed, the calculation
of the kite’s angular rate when applying a steering input shows
that the yaw rate is the parameter the most influenced by the
input. The others will be neglected as their contributions are
sufficiently small compared to the yaw rate. An empirical
equation discussed in other works [16], [11] and [9] is pro-
tosed to account for this behavior. The dynamic response to
a steering input \( \delta \) is thus modelled by the following equation:

\[
\dot{\psi} = g_k v_a \delta + M \cos \theta \sin \psi \frac{v_a}{v_a},
\]

(9)

where \( v_a \) is the airflow projected on the \( x_b \) axis and \( g_k \)
is a constant parameter that must be deduced from real
measurements. As explained in [16] the second term of the
equation accounts for the effect of the gravitational force,
\( M \) being another constant to determine. It is noteworthy to
understand that depending on the sign of \( M \), a kite behavior
can differ. Indeed, if \( M \) is positive, without any steering input
nor being aligned with the wind, the kite has a tendency to
fall towards the ground. If \( M \) is negative, the kite moves
towards the zenith and aligns itself with the wind.

While \( \psi \) is the angle of the kite heading relative to \( x_b \), its
yaw rate \( \dot{\psi} \) calculated with relation (9) only takes into
account the angular velocity of the kite reference frame \( R_b \)
relative to the kite position reference frame \( R_{ko} \). However,
as the kite is evolving on a sphere, the actual yaw rate seen
by the ground reference can be defined by the relation:
\( \dot{\psi} = \dot{\psi} + \dot{\psi}/R_{kn} \). The second term is equal to
\( -\phi \sin \theta \). Finally, the turn rate law used in the model is:

\[
\dot{\psi} = g_k v_a \delta + M \cos \theta \sin \psi \frac{v_a}{v_a} - \phi \sin \theta.
\]

(10)

2.2.6 State space representation

Using relations (2), (3), (4) and (10), the model can be written
as a state space representation with 5 states. The input
\( U \) is a vector with the asymmetrical steering input \( \delta \) and
the symmetrical steering input \( \epsilon \).

\[
\begin{align*}
\dot{X}(t) &= f(X(t), U(t)) \\
Y(t) &= g(X(t), U(t))
\end{align*}
\]

with \( X = \begin{bmatrix} \theta \\ \phi \\ \dot{\theta} \\ \dot{\phi} \\ \psi \end{bmatrix} \) and \( U = \begin{bmatrix} \delta \\ \epsilon \end{bmatrix} \).

(11)

f is a nonlinear function, gathering the equations (2), (3), (4)
and (10) that describes the evolution of the system state \( X \),
while \( g \) is the output function. A few parameters are still
unknown before a simulation of this model can be achieved: \( g, M, C_D, C_L \), and \( C_t \). They depend on the kite wing itself, and
need to be determined by experimental test as explained in the
next section.

3 EXPERIMENTAL DATA

3.1 EQUIPMENT

In this section, we present how the experimental data were
recorded by Beyond the Sea. The kite used is all experiments
afterwards is a 15 m² kite, with 3 tethers of 25 m. The exper-
iments were made on the ground. The anchor point consists
of a manual steering device that allows for asymmetrical and
symmetrical steering by acting on two handles at the end of the
two steering lines. The traction line is connected to the
ground as well, but is not used for control. On the wing, the
measurements are made with an inertial measurement unit,
which will be referenced as IMU. The IMU is attached at
the center of the leading edge of the kite, and communicates
with the ground via wireless communication. On the ground,
each tether is connected to a dynamometer to measure the
sustained force. The IMU communicates a reference frame
which can indicate the kite attitude at each time. Additionally,
an anemometer measures the true wind speed and direction.

The kite is steered manually, via the two steering lines. The
sensors enable the measurement of the kite attitude, the teth-
ers force and their steering. It enables the computation of all
necessary signals to determine the model parameters defined
in section 2.

3.2 TURN RATE COEFFICIENTS

3.2.1 Slacking and hauling

To deduce the kite position in the wind sphere using the IMU,
it becomes necessary to correct the slacking and hauling of
the kite. Indeed, the position is estimated by calculating the
attitude of the IMU. It would be sufficient if the kite plane was
always tangent to the wind sphere. However, it is not the case,
as the wing possesses an incidence due to the symmetrical
steering input. Indeed, the steering being manual, it can be
quite difficult to ensure the slacking and hauling to be null.
Once this effect is corrected, the real position of the kite is
calculated as explained above, as if it had no slacking and
hauling.

The relation between the symmetrical steering and the in-
cidence of the kite is linear and can be written as follow:

\[
\alpha_i = K_i \epsilon + \epsilon_0,
\]

(12)

where \( \epsilon_0 \) is an offset, and \( K_i \) is a linear coefficient. The
purpose is to be able to deduce the incidence \( \alpha_i \) applied to
the kite, knowing the symmetrical input. By knowing this
data, it becomes possible to compute the real kite position on
the wind sphere. To deduce this coefficient, an experimental
Figure 3: Measured data of incidence as function of a symmetrical input, to determine the $K_i$ coefficient

measure was made, where the kite was held in position near the ground. Its only degree of freedom was through slacking and hauling, which allows to find the relation between the steering input and the resulted incidence.

Figure 3 illustrates the linear fitting of the measured data of incidence, thanks to the IMU. From this, $K_i = 0.44$. Then, knowing the symmetrical input $\epsilon$ of the steering lines, the incidence generated at any time can be directly calculated by (12), representing the slacking and hauling of the wing.

3.2.2 Turn rate law coefficients
In order to compute the coefficients of the turn rate law (10), experimental data from a dynamic flight around the zenith have been analysed. In this case, the asymmetrical steering input $\delta$ is the most important to deduce the two parameters $g_k$ and $M$ as equation (10) is directly connected to $\delta$. Experimental measures allows the computation of the kite position $\theta$ and $\phi$, as well as the kite heading $\psi$. However, the actual yaw rate measured by the IMU is $\dot{\psi}_{\text{eq}}$. Therefore, the objective is to compare the measured $\dot{\psi}$ with the reconstructed $\dot{\psi}$, noted $\dot{\psi}_{\text{recon}}$, using the two parameters and equation (9). Using a least squared algorithm, the two parameters $g_k$ and $M$ can be computed, so that the yaw rates are comparable.

Equation (12) allows the correction of the kite position, by adjusting the slacking and hauling $\alpha_i$, induced by the symmetrical steering input $\epsilon$. Then, the parameters $g_k$ and $M$ are deduced as explained above:

$$g_k = 0.15 \text{ and } M = 3.9.$$ 

(13)

Figure 4 shows the reconstructed yaw rate based of the two parameters.

3.3 AERODYNAMIC COEFFICIENTS
The others parameters of the model that need to be found are the aerodynamic coefficients. Those are function of the angle of attack $\alpha$. In the literature [8], [12], the aerodynamic profile used are often similar to airplane wing determined by experiments and some simplifications [23] or specific to the wing used. Therefore, an estimation method of those parameters based on experimental data is proposed here, specific to the kite used by Beyond the Sea during experiments.

3.3.1 Assumptions
The experimental test conditions to this end are particular, as the kite is placed in front of the true wind. Therefore, the azimuth $\phi$ is null and the kite evolves in straight line, controlled by the slacking and hauling of the kite. Thus, two assumptions can be made:

- first, the heading $\psi$ is already aligned with the wind. This allows the transverse aerodynamic force (7) to be neglected.

- secondly, the force sustained by all tethers is measured by three dynamometers. Thus, the total traction is known. Then, the kite speed while going towards the zenith must be low enough so that the total traction can be approximated equal to the opposite of the aerodynamic force from equation (6).

3.3.2 Drag and lift coefficient

Figure 5: Drag coefficient $C_D$ (red line) and lift coefficient $C_L$ (blue line) from experimental data

Based on the two assumptions above, the drag and lift forces are calculated, as well as the angle of attack $\alpha$ of the kite, while the kite is steered gradually by a slacking and hauling input $\epsilon$. Thus, the kite will move towards the zenith and away from it, while remaining aligned with the true wind. Then, by computing the drag and lift as functions of the angle of attack $\alpha$, the drag and lift can be estimated. An example is shown in Figure 5 during a descent of the kite. The stall is visible from the lift coefficient around an angle of attack of $40^\circ$. However, due to unstable test conditions, the recurrence of those estimations is not perfect yet. For this reason,
future experimental tests are planned to remake aerodynamic coefficient estimations on the kite. Concerning the transverse aerodynamic force, no estimation of that coefficient was made yet. However, it was taken sufficiently big so that, in dynamic motion, the kite remains aligned with its airflow, as the experimental tests suggest. The kite model can now be implemented and simulated on different configurations.

4 “8” SHAPE TRAJECTORY

4.1 PATH-FOLLOWING STRATEGY

4.1.1 Desired trajectory

The actual purpose of Beyond the Sea is to develop a controller that steer the kite on a desired trajectory. It has been found that the optimal trajectory to generate maximum traction is an “8” shape trajectory. It also hold the advantage of not twisting the lines.

Geometrically, there are several ways to describe an “8” shape. A lemniscate of Bernoulli was chosen as the desired trajectory for its interesting properties, such as gradual variation of its curvature radius. Such a trajectory can be defined by its azimuth and elevation with the parameter $s$:

$$
\begin{align*}
\phi &= \phi_0 + \Delta\phi \frac{\sin s}{1 + \cos^2 s}, \\
\theta &= \theta_0 + \Delta\theta \frac{\sin s \cos s}{1 + \cos^2 s},
\end{align*}
\quad s \in [0, 2\pi].
$$

(14)

Other parameters enables to describe the trajectory at any position or rotation. As show in Figure 6, once the lemniscate is parameterized, it can be projected into the quarter sphere.

Figure 6: Lemniscate of Bernoulli projected in wind sphere.

As equation (14) suggests, the trajectory is not a function of time. Therefore, following the path does not depend on time but on space. The trajectory is thus spatially discretized in a series of points $P_i$, evaluated in the $R_{WR}$ reference frame.

4.1.2 Proposed algorithm

The objective is obviously for the kite to reach the trajectory, if it is not already the case, and then to be able to follow it. Therefore, the path following algorithm needs to be able to reduce the distance and align the kite with the trajectory. Every time step, the algorithm will find which point of the trajectory the kite needs to follow. It will be referred as $P_i$. The distance between the two is then calculated and noted $d$.

Section 2.2.4 introduced the difference between the kite heading angle and the kite velocity angle. To align the kite trajectory with the desired trajectory, the kite velocity angle $\chi$ needs to be collinear with the tangent at point $P_i$. Thus, to control the kite trajectory means controlling its velocity angle and comparing it to the desired trajectory. Hence by choosing an appropriate kite velocity orientation $V_{cmd}$, it is possible to reduce the distance and align the kite at the same time.

4.2 PATH-FOLLOWING ALGORITHM

4.2.1 Distance to trajectory and reference point

The principle of the algorithm is to define a reference frame $R_t = (x_t, y_t, z_t)$ at the next waypoint of the trajectory, hence $P_{i+1}$. At first, the algorithm does not know which point is the closest. An initial point $P_i$ must be chosen, for instance it can be the first point of the trajectory, for $s = 0$. Then, the kite position is evaluated from $R_{WR}$ to this new trajectory reference frame $R_t$ at $P_{i+1}$.

Initial point $P_i$

$P_{i+1}$

$P_{i+2}$

Figure 7: Determination of actual reference point and distance.

Then, as shown in Figure 7, the sign of the kite x-coordinate relative to $R_t$ informs if the kite went past point $P_{i+1}$. If it did, then the reference point is moved to $P_{i+1}$ and a new reference frame is computed at $P_{i+2}$. Hence if the kite did not pass the point $P_{i+1}$, it means the kite is between points $P_i$ and $P_{i+1}$. The y-coordinate of the kite position immediately gives the distance $d$. Depending on the initial point, the algorithm might pass several points until it reaches a point where the correct conditions are met.
Figure 8: Simulations with various desired trajectories. Wind of 40 km.h\(^{-1}\), tether length of 50 m and kite of 15 m. The kite velocity vector is represented by the black arrow, while the kite heading is represented by the green arrow.

4.2.2 Desired orientation

The kite position relative to the trajectory point \(P_i\) being known, its orientation angle also needs to be computed. As written previously, the kite velocity angle \(\chi\) is used to control the kite direction. Therefore, a desired velocity angle has to be calculated, noted \(\chi_{\text{cmd}}\), which will be compared to the actual \(\chi\) afterwards. The objective is to compute \(\chi_{\text{cmd}}\) so that the kite reaches the trajectory and aligns with it at the same time:

\[
\chi_{\text{cmd}} = \chi_C + \Delta \chi_{\text{cmd}}(d). \tag{15}
\]

The tangent at point \(P_i\) is computed first, as it represents the actual desired direction that the kite needs to be aligned with. The first term of equation (15) is the angle between the tangent translated at the kite position and its current geodesic, noted \(\chi_C\) and adds the information of the path orientation as shown in Figure 9 and depicted in [10].

If \(\Delta \chi_{\text{cmd}}(d)\) is not added in relation (15), the kite steers to have the same alignment but is not necessarily on the path itself. Hence another term is necessary to bring the kite closer to the trajectory: \(\Delta \chi_{\text{cmd}}(d)\) is defined by

\[
\Delta \chi_{\text{cmd}}(d) = \arctan\left(\frac{d}{d_0}\right). \tag{16}
\]

Equation (16) depicts the angle added to tangent direction \(\chi_C\), which is a function of the distance \(d\). This angle brings the kite closer to the trajectory. If the kite is far from the trajectory, then the kite will go directly towards the point \(P_i\), perpendicular to the tangent. Hence \(\Delta \chi_{\text{cmd}} = +\frac{\pi}{2}\). If \(d\) is relatively small, then the alignment is more important. Hence \(\Delta \chi_{\text{cmd}} = 0\).

That is exactly the role of the parameter \(d_0\) in (16), which decides how the kite needs to be oriented relative to the point \(P_i\) in function of the kite distance \(d\). This parameter can be chosen static, or dynamically relative to the kite speed.

4.2.3 Closed loop

According to the previous comments, the desired velocity angle can be compared to the actual kite velocity angle. This angle difference is used in the sequel to define a trajectory control loop. In this loop, the kite model determine \(\chi\), that is compared to \(\chi_{\text{cmd}}\), which depends on the desired trajectory and the kite position.

Then, a simple proportional controller is used at first, to ensure the closed loop stability and obtain a sufficient performance level for modelling and path following. However, a CRONE [14] [18] controller, a robust controller based on fractional differentiation [21] [19] will be discussed and justified by authors in a future publication dedicated to control. To sum up, the robustness will permit to ensure good performance for a large set of wind-speeds and to take into account both the uncertainty and nonlinear behavior of the model of the controlled system.
4.3 SIMULATIONS

The model described in section 2, the parameters determined in section 3 and the path-following strategy explained in section 4 are then implemented using MATLAB/SIMULINK software to simulate the closed loop.

As explained in Section 1, hydraulic drives will be used to steer the lines. In the following simulations however, the dynamical model of the drives has been omitted to let the closed loop steer the lines without any constraint. Then, by studying the simulated constraints on the command \( \delta \) needed for steering as depicted in Figure 10, actuators that can withstand those will be chosen accordingly.

Figure 10: Command \( \delta \) for simulation 8(a).

Figure 8(a) displays a simulation with a trajectory centred in the wind window. As expected, the kite follows the desired trajectory correctly, and its transition from its initial position and the desired trajectory is also gradual. The slacking and hauling were not touched. The command \( \delta \), periodic with the same period as the trajectory, ranges from \(-1\text{m}\) to \(1\text{m}\), as depicted in Figure 10. The noise on the command is due to the lack of a low pass effect in the controller and the lack of actuators that would act a low-pass filter.

The path following strategy works well for any trajectory that is within the wind window. Indeed, Figure 8(b) shows the kite follows a non-centred path correctly. In both simulations, computing the angle of attack \( \alpha \) shows that in dynamic flight and while following an “8” path trajectory, variations of \( \alpha \) are relatively small. Thus, the kite should not stall during that kind of motion.

Computing the generated traction for each simulation also reveals the efficiency of any trajectory. As anticipated, simulation 8(a) provides more traction than simulation 8(b), as its path is directly in the most powerful zone of the wind window. The comparison is displayed in Figure 11.

5 CONCLUSION

In this paper, a kite point-mass model is developed, in which a transverse aerodynamic force, as well as slacking and hauling have been taken into account. Then, based on experimental data provided by Beyond the Sea, all parameters of the model were characterized. A path-following strategy was devised so that the kite can follow an “8” shape trajectory. Thereafter, simulations made it possible to estimate actuators constraints and compare the generated traction between different trajectories.

Future work will be oriented on the design of a robust controller using a CRONE approach, as well as a periodic analysis of the linearized model around an “8” shape trajectory. This study will lead to a more theoretical method that will justify the robust controller design.

Also, future experimental tests will enable refining the parameters of the model to obtain more realistic simulations. This model will be used to design a new robust controller whose efficiency could be evaluated in experimental dynamic flight. Additionally, a static flight algorithm is being designed, in order to improve the quality of the transitions between static and dynamic flight.

6 ACKNOWLEDGMENTS

This work was granted by the French Environment and Energy Management Agency (ADEME), the french National Association for Research and Technology (ANRT), and the Nouvelle-Aquitaine Regional Council.

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7 AUTHORS BIOGRAPHY

Baptiste CADALEN was born in Brest, France, on July 24, 1991. He graduated from the Ecole nationale superieure d’electronique, informatique, telecommunications, mathematiques et mecanique de Bordeaux (ENSEIRB) in 2014. He holds the current position of Ph.D. student at the IMSc laboratory and the Beyond the Sea society, led by Yves Parlier. He is responsible for the modelling and robust control of the kite wing in order to create an automatic pilot aiming to provide ship with an alternative source of traction.

Patrick LANUSSE was born in Bordeaux, France, on January 12, 1966. Since 1990 he has been with the CRONE team of the IMS laboratory where he has been working on Robust Control, Fractional Order Controllers and more particularly on CRONE Control. He received his Ph.D. degree in 1994 by proposing to use the complex order fractional operator to design robust control-systems. Since 1995 he has been Associate-Professor of Control Theory at the Bordeaux Institute of Technology. His research is about CRONE control to increase performance without decreasing robustness. He developed the control-system design CRONE Toolbox for Matlab which can be downloaded freely since 2010.

Jocelyn SABATIER graduated from the Ecole Nationale Superieure des Arts et Metiers (ENSAM) in 1992. He received the Ph.D. degree in Control Theory from Bordeaux I University in 1998. He is the author and co-author of about 120 publications. He is presently Professor at University Institute of Technology (IUT) of Bordeaux in the fields of control and electrical engineering. His current research interests are in the area of fractional systems through activities in simulation, systems theory and distributed parameter systems, and in the area of control theory through extensions of CRONE control to time varying systems and nonlinear systems.