VELOCITY PREDICTION OF WING-SAILED HYDROFOILING CATAMARANS

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The paper presents a Velocity Prediction Program for hydrofoiling catamarans with solid wing sails. Starting from a description of the mechanical model, suitable models are identified for the forces that act on the boat components. The study is deliberately limited to means for restricted methods where computational resources and budgets are limited. Enhanced lifting line approaches are described for the wingsail and appendages. Windage is calculated from force coefficients and dynamic pressure while hull resistance is determined by means of a potential flow solver. The description of the implementation is followed by the presentation of the results including a comparison to measured data. Additionally significant findings in terms of overall force composition and distribution as well as loading on specific components of the catamaran are presented and discussed. Finally, some of the challenges encountered during the study are discussed. It was found that the VPP predictions compared favourably with measured data from the 34th America’s Cup in San Francisco in 2013.

NOMENCLATURE

\begin{align*}
F & \quad \text{Force [N]} \\
M & \quad \text{Moment [Nm]} \\
c_F & \quad \text{Coefficient of force [-]} \\
c_L & \quad \text{Coefficient of lift [-]} \\
z & \quad \text{Vertical distance [m]} \\
z_C & \quad \text{Vertical collocation point position [m]} \\
\rho & \quad \text{Density of water/air [kg/m}^3]\text{]} \\
AWS & \quad \text{Apparent wind speed [m/s]} \\
BS & \quad \text{Boatspeed [m/s]} \\
TWA & \quad \text{True wind angle [°]} \\
TWS & \quad \text{True wind speed [m/s]} \\
A & \quad \text{Reference area [m}^2]\text{]} \\
AoA & \quad \text{Angle of attack [°]} \\
\Gamma & \quad \text{Vorticity []} \\
v_{\text{indC}} & \quad \text{Velocity induced at point } P_c \text{ [m/s]} \\
u & \quad \text{Incident flow velocity [m/s]} \\
v_{\text{i,j}} & \quad \text{Induced velocity vector [m/s]} \\
v_{\text{mi,j}} & \quad \text{Mirror image induced velocity vector} \\
\hat{\mathbf{w}} & \quad \text{Total velocity vector [m/s]} \\
\hat{\mathbf{v}} & \quad \text{Total induced velocity vector [m/s]} \\
s_i, s_i & \quad \text{Local span/span vector [m]} \\
c_i, c_i & \quad \text{Local chord/chord vector [m]} \\
\hat{r} & \quad \text{Vector between start of free vortex filament and collocation point [m]} \\
k & \quad \text{Vortex strength reduction factor [-]} \\
d & \quad \text{perpendicular distance between vortex filament and collocation point [m]} \\
d_{\text{core}} & \quad \text{diameter of viscous vortex core [m]} \\
Re & \quad \text{Reynolds number [-]} \\
Rt & \quad \text{Total resistance [N]} \\
Rf & \quad \text{Frictional resistance [N]} \\
Rw & \quad \text{Wave-making resistance [N]} \\
Rr & \quad \text{Residuary resistance [N]}
\end{align*}

1 INTRODUCTION

A lot of the recent developments in high performance yacht design have focused on hydrofoiling and rigid wing sails. These technologies have been employed separately in sailing yachts for a long time, but were rarely combined in one boat. This eventually changed prior to the 34th America’s Cup in 2013, which was contested in catamarans equipped with wings and appendages designed to promote foiling.

Similarly velocity prediction programs (VPPs) are widely used tools among yacht designers for the assessment of the speed potential of yachts. As a result of extensive development conducted during previous years, when the most prestigious regattas were sailed in monohulls, VPPs concerning this type of boat have become quite mature. However, experience shows that every VPP needs some degree of adaption to the type of boat under consideration. As multihulls have rarely gotten the attention of a wider audience prior to the 34th America’s Cup, comparatively little work has been published on velocity prediction of catamarans. This effect is amplified by the fact that, because an accurate VPP represents a competitive advantage, sailing teams and design offices mostly keep their performance and VPP knowledge a secret.

2 FORCE DECOMPOSITION

The fundamental principle of each VPP is the equilibrium of forces and moments, as expressed in Eq.(1).

\[ \sum F = \sum M = 0 \] (1)

Figure 1 illustrates the force decomposition applied in this study. The aerodynamic force is split into windage and rig forces in the longitudinal and transverse
directions. The hydrodynamic forces are assembled from the contributions of the hull and appendages. The hull is affected by buoyancy and resistance while the appendages exert forces along all three axes on the boat.

As shown in figure 1 the catamaran is modelled with the windward hull just above the water surface, since the righting moment is at its maximum in this state.

3 AERODYNAMIC FORCES

3.1 WINDAGE

Windage forces are determined as the product of dynamic pressure and force coefficient (Eq. (2)), which is varied depending on the geometric characteristics of the object.

\[ F_i(z_i) = c_{Fi} \frac{\rho}{2} A WS(z_i) A_i \]  

(2)

Windage items have been divided into three groups for this study. Cables and parts with circular cross sections and high L/D ratios operate in laminar flow [1]. These have been vertically segmented to take changes in apparent wind over height into account. Platform components like hulls and beams extend mainly parallel to the water surface and are thus subject to a constant apparent wind speed and direction. Since the incident flow, dimensions and orientations of the remaining objects like the crew, winches and media equipment are somewhat unknown, they are modelled as 10% of the platform drag. Table 1 lists the groups and the associated force coefficients.

<table>
<thead>
<tr>
<th>Group</th>
<th>Description</th>
<th>( c_f ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>cables</td>
<td>Circular cross sections &amp; high L/D ratios (2D laminar flow)</td>
<td>1.1</td>
</tr>
<tr>
<td>platform</td>
<td>Parallel to waterplane (constant apparent wind)</td>
<td>0.6</td>
</tr>
<tr>
<td>misc.</td>
<td>Unknown remaining items</td>
<td>10% platform drag.</td>
</tr>
</tbody>
</table>

3.2 WING FORCES

GRAF et al.[2] have shown that the forces developed by a wingsail can be accurately modelled by a modified lifting line method, which is briefly summarized below.

A bound vortex filament is created by expanding the lifting vortex from the Kutta-Joukowski theorem along the span of the wing. To comply with Thompson’s law, the vortex filament is extended to infinity along the incident flow at the wing tips, forming a so-called horseshoe vortex.

Any change in lift can be represented by modifying the strength of the bound vortex and associated shedding of the differential vorticity through a free vortex filament. A vortex sheet is created by placing an infinite number of free vortex filaments between two reference points.

According to the Biot-Savart law, shed vorticity induces a velocity normal to incident flow and span that reduces the effective angle of attack. Figure 2 illustrates the discretization of the wing into panels.

The total velocity induced in collocation point \( P_e \) can be calculated using Eq. (3) [2]. Note that the strength of the root vortex is dampened by a factor \( EP \) to model end plate effects of the platform.

![Figure 1 Force modelling of the catamaran](image1)

![Figure 2 Wing discretization](image2)
\[ v_{\text{ind}Cj} = \sum_{i=1}^{N} - \frac{1}{4\pi} \left( \frac{\Gamma_1 - \Gamma_{i-1}}{z_i - z_{i-1}} \right) \ln \left( \frac{z_i - z_{Cj}}{z_{i-1} - z_{Cj}} \right) \]
\[ - EP \frac{\Gamma_0}{4\pi (z_0 - z_{Cj})} + \frac{1}{4\pi} \frac{\Gamma_N}{(z_N - z_{Cj})} \] (3)

Vorticity is derived from Eq. (4).
\[ \Gamma = \frac{u}{2} * A * c_L \left( AoA - \frac{v_{\text{ind}}}{u}, Re \right) \] (4)

Since Eqs. (3) and (4) depend on each other, the solution has to be found iteratively, for which under-relaxation is applied [2]. Once the effective angle of attack has been determined it can be used to look up the corresponding profile drag in a table.

4 HYDRODYNAMIC FORCES

4.1 APPENDAGE MODEL

The appendages have been modelled using a three dimensional variant of the lifting line method. A wing of arbitrary orientation is discretized with horseshoe vortices according to figure 3.

Figure 3 Discretization of appendages

PHILLIPS and SNYDER [3] derived the induced velocity vector generated by a horseshoe vortex positioned along the \( \frac{1}{4} \)-chord line as Eq.(5).
\[ \bar{v}_{i_{n-1}} = \frac{\Gamma_{n-1}}{4\pi} k_2 \left( \frac{B_S}{B_S} \times \bar{r}_2 \right) \]
\[ \frac{|\bar{r}_2|}{|\bar{r}_2| - \left( \frac{B_S}{B_S} * \bar{r}_2 \right)} \]
\[ + k_b \left( \frac{|\bar{r}_1| + |\bar{r}_2|}{|\bar{r}_1| |\bar{r}_2|} \right) \left( \frac{|\bar{r}_1| + |\bar{r}_2|}{|\bar{r}_2|} \right) \]
\[ - k_1 \left( \frac{B_S}{B_S} \times \bar{r}_1 \right) \frac{|\bar{r}_1|}{|\bar{r}_1| - \left( \frac{B_S}{B_S} * \bar{r}_1 \right)} \] (5)

The factors denoted \( k \) in Eq.(5) above has been introduced to avoid the singularity in the Biot-Savart law through gradual reduction of the vorticity for vortex distances approaching zero. This effect enables a finer discretization of the wing to offset the reduced accuracy from bound vortices of constant strength compared to using vortex sheets as done for the wingsail. \( k \) is computed according to Eq. (6) from the perpendicular distance between vortex filament and collocation point \( d \) and the size of the viscous vortex core \( d_{\text{core}} \) [4]. Values of \( d_{\text{core}} = 0.01 \text{ m} \) and \( m = 2 \) were selected for this study.
\[ k = \frac{d^{m}}{(d_{\text{core}}^m + d^{2m})^{\frac{1}{m}}} \] (6)

Vorticity is calculated according to Eq.(7).
\[ \Gamma_1 = \frac{0.5 * B S^2 * s_1 * c_1 * c_{b1}}{|\bar{w}_i \times \bar{c}_i|} \] (7)
The total velocity is the sum of incident flow and induced velocity as shown in Eq. (8).
\[ \bar{w}_i = B S + \bar{v}_i \] (8)
The angle of attack is determined as per Eq. (9) in which \( q_i \) and \( c_i \) denote the vectors normal to the wing surface and along the chord, respectively [3].
\[ AoA_i = \tan^{-1} \left( \frac{\bar{w}_i * \bar{q}_i}{\bar{w}_i * \bar{c}_i} \right) \] (9)

Summing up the individual contribution of all horseshoe vortices and their mirror images reflected at the water surface gives the total induced velocity, Eq.(10).
\[ \bar{v}_i = \sum_{j=1}^{n} \bar{v}_{i_{j}} + \sum_{j=1}^{n} \bar{v}_{m_{i_j}} \] (10)
Emerged wing sections are modelled by setting their associated density to zero.

4.2 APPENDAGE DESIGN

The appendages have been modelled according to figure 4. The winglet or tip section is joined to the daggerboard by a radial transition. The rudder assembly consists of two perpendicular wings. Although inspiration was drawn from the early AC72 daggerboard designs, the authors do not claim these designs to be particularly sophisticated.
The selected dimensions are given in table 2.

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Value [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>daggerboard chord</td>
<td>0.80</td>
</tr>
<tr>
<td>rudder chord</td>
<td>0.30</td>
</tr>
<tr>
<td>elevator chord</td>
<td>0.25</td>
</tr>
<tr>
<td>daggerboard span</td>
<td>3.50</td>
</tr>
<tr>
<td>transition segment</td>
<td>0.39</td>
</tr>
<tr>
<td>winglet span</td>
<td>1.25</td>
</tr>
<tr>
<td>rudder span</td>
<td>2.25</td>
</tr>
<tr>
<td>elevator span</td>
<td>1.25</td>
</tr>
<tr>
<td>daggerboard cant</td>
<td>10.00</td>
</tr>
<tr>
<td>winglet dihedral</td>
<td>90.00</td>
</tr>
</tbody>
</table>

4.2 HULL RESISTANCE

Following the suggestion of OLIVER [6], the wavemaking resistance of the hull is determined from the MICHELL formula, which is implemented into the MICHLET research code [7]. It is based on slender ship theory and is thus ideally suited for analysing sailing catamaran hulls. Frictional resistance is calculated as the product of static wetted surface area, dynamic pressure and friction coefficient derived from the ITTC’57 friction line.

The resistance predictions have been compared against towing tank test data of a TORNADO class hull. The results are presented in figure 6, which shows resistance as a function of Froude number. It can be seen that there is good agreement between both sets of data up to a Froude number of about 0.8. The wavemaking resistance hump around Fn = 0.4 is well captured. The increasing disparity can be explained by the lack of viscous pressure resistance in the prediction data. Since viscous pressure resistance is caused by friction, it is expected to be approximately proportional to frictional resistance.

As a side note, the computations referred to above were carried out on a laptop in less than 20 seconds.
5 IMPLEMENTATION

The VPP has been implemented as a constraint optimization routine in MATLAB. As such, it triggers function calls to determine the forces acting on each component for the current set of variables and only falls back to classic interpolation to obtain hull resistance and hydrostatics.

Forces and Moments are evaluated for all six degrees of freedom. Equilibrium is enforced for all components of translation and the moment about the vertical axis. Upper and lower limits are put in place for the heeling and trimming moments. The heeling moment boundaries result from the transversely moveable (crew) weight and the assumption of the windward hull being just clear of the water. The trimming moments are restricted by the positions of the longitudinal centre of buoyancy for allowed trim extrema of ±1.5°. Trim is not explicitly modelled, but is taken into account by varying the rudder elevator angle of attack, which then attains the meaning of a target trim plus elevator angle. As the trimming moment limits depend on the hydrostatic buoyancy force, full equilibrium is enforced for the moment about the transverse axis for foiling states.

In total the VPP optimizes eight variables, which are listed in table 3 with their corresponding bounds. Twist is defined as angle differential between the root and tip sections of the wing and varies linearly in-between.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>min.</th>
<th>max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>boatspeed</td>
<td>[m/s]</td>
<td>0</td>
<td>Infinity</td>
</tr>
<tr>
<td>sheeting angle</td>
<td>[°]</td>
<td>-10</td>
<td>Infinity</td>
</tr>
<tr>
<td>wing twist</td>
<td>[°]</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>rake</td>
<td>[°]</td>
<td>-10</td>
<td>10</td>
</tr>
<tr>
<td>leeway angle</td>
<td>[°]</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>rudder angle</td>
<td>[°]</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>Elevation $z_o$</td>
<td>[m]</td>
<td>0</td>
<td>2.45</td>
</tr>
<tr>
<td>trim/elevator angle</td>
<td>[°]</td>
<td>-1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The rig was originally modelled as a single element wing and without the jib in this study due to a lack of data and to reduce complexity. However, GRAF et al. [5] found that the jib does add little if any additional driving force to the wing system for upwind sailing cases. Furthermore, COLLIE et al. [8] stated that a multi-element wing was only superior in (light-wind) downwind sailing. Therefore, accurate force predictions can still be expected at least for upwind and reaching conditions from this much simplified model.

Lift and drag coefficients for the wing models described are supplied in tables as functions of angle of attack and Reynolds number. As lifting line theory is based on the assumption of two dimensional flows, it is not suitable for modelling of separated flow which exhibits distinctive three dimensional features. To force the optimizer into the attached flow regime, the lift curves have been modified as shown in figure 7 to impose a higher penalty on stalled flow while retaining the curve smoothness necessary for stable interpolation.

6 RESULTS

Figure 8 shows a polar plot of the velocities calculated by the VPP for true wind speeds (TWS) between 5.0 m/s and 12.5 m/s. It can be seen that speeds generally increase with the wind angle and wind speed as would be expected. Peak speeds are reached at a wind angle of about TWA = 120°. The transition to foiling can be spotted as a sudden distinctive increase in speed for all but the lowest windspeed. The angle for the beginning of foiling varies between TWA = 60° and about TWA = 75° with the lowest angle found for a windspeed of TWS = 10.0 m/s. It is not quite clear why the inception of foiling then goes back towards a higher angle for TWS = 12.5 m/s. Due to this anomaly the potential boatspeed for the windspeed of 10.0 m/s is higher than for TWS = 12.5 m/s.

The dotted lines show the results for running the VPP in reverse, i.e. from large to small wind angles. It can be seen that around TWA = 60° higher boatspeeds can be achieved when turning upwind from higher wind angles. This is attributed to the fact that the yacht can stay on its foils once the drag of the hull has been overcome once.

It can also be noticed that the lines plotted in the polar end at different true wind angles. This is due to the fact that the VPP has been run until no feasible solution was found anymore, which means that the optimizer was not able to find a solution that fulfils all equilibrium constraints. For downwind sailing this typically occurs when the minimum heeling moment required to lift the windward hull out of the water cannot be reached.
Figure 8 AC72 velocity polar for windspeeds of 5.0 m/s (black), 7.5 m/s (green), 10.0 m/s (red) and 12.5 m/s (blue)

Figure 9 shows a comparison between the data recorded race 5 of the 34th America’s Cup [9] and the predictions of the VPP. Race 5 has been selected since the deviations from the average windspeed of about 20 knots have been found to be relatively small. It has to be kept in mind that in order to display the recorded data, it has been processed by selecting the highest boatspeed for a range of 1.0° around whole degrees. Hence, there is a good chance that momentary peak speeds caused by gusts or GPS measurement errors are contained in the data. Additionally the recorded speeds include the effects of mass momentum, which is evident at the zero crossings where speed was carried through tacks and gybes. In summary this data represents the upper limit of attainable boatspeed. However, a qualitative analysis still seems to be reasonable. It can be seen that upwind performance seems to be severely under-predicted. From the inception of foiling at TWA = 60° to about TWA = 85°, however, the prediction agrees well with the recorded data. Continuing further towards broad reaching, the recorded speeds stay well below VPP predictions. It has to be pointed out, though, that the yachts did not sail at these angles for long times during the race, as the course called for upwind and downwind VMG sailing after the reaching start. The greater scattering of the data in this area also indicates low emphasis on maximum speed at these angles. Towards the maximum downwind VMG angle the scattering reduces again and the agreement between the datasets improves.

Figure 9 Comparison of recorded performance data (ETNZ race 5) to velocity predictions

Figure 10 shows wing lift coefficient at TWA = 60° plotted over height for the windspeeds known from the polar diagram. It was expected that the information could shed some light on why the foiling inception angle differs between the different wind velocities. It can be seen the maximum lift is always produced about 5.0 m above the wing root and then decreases with height. For the higher wind velocities it actually becomes negative which means righting moment is being generated by the wing. It has to be noted that for at wind speeds of 7.5 m/s and 10.0 m/s the root section appears to be stalled. This is a result of the relatively low stall angle of the single element wing without jib. Usually effort is put into avoiding stall since it is associated with high drag. In this case however, it seems to be beneficial as the boat starts to foil earlier with a stalled root section at a TWS of 10.0 m/s than with the root section operating at a high lift coefficient at TWS = 12.5 m/s. Now this is probably an error in the modelling of the wing aerodynamics, but it hints at the cause of why the boat starts foiling earlier at a lower wind velocity. The root of the wing sees the smallest apparent wind angles and therefore produces high lateral force which has to be balanced by the appendages. This increases the induced drag of daggerboard and rudder, which might prevent foiling.
A non-quantifiable result of this study was the observation of the stability of the VPP being influenced by the cant angle of the daggerboard. It was initially assumed that there is no difference in the optimization between a vertical daggerboard and one canted inwards by 10°. However, it was found that the VPP was finding equilibrium states much faster and more reliably with the canted board. This represents a nice analogy to what sailors and designers had experienced during the design of the actual daggerboards.

One of the challenges encountered during this study was the large number of variables to be optimized. The selection of variables has been a trade-off between realistic modelling and the ability to optimize reproducible parameters in a sensible amount of time. The number of adjustable parameters has been reduced considerably from what is found on actual boats. Examples of parameters that have remained fixed are daggerboard immersion and cant angle plus wing camber and trim angle, camber and twist of the headsail which has not been modelled.

8 CONCLUSIONS

- A VPP for performance prediction of wing-sailed catamarans was successfully developed.
- Predicted resistance data agreed well with experimental data for a Tornado hull.
- Enhanced lifting line approaches were used for the wingsail and appendages.
- Performance predictions compare reasonably well to measured performance data.
- The transition between floating and foiling states has been captured.
- The VPP requires little resources in terms of software and hardware, and ran quickly on a laptop.

REFERENCES

AUTHOR’S BIOGRAPHIES

N. Hagemeister currently works in the fluid dynamics department of Van Oossanen Naval Architects B.V, where he is in charge of fluid dynamic analysis and optimization of commercial and recreational vessels. He has previously been a wind tunnel manager with Yacht Research Unit Kiel and an intern with Incidences Sails La Rochelle. He graduated with M.Eng from University of Applied Sciences Kiel on the topic of velocity prediction of hydrofoiling catamarans. He spent 6 months during 2014-2015 in the Yacht Research Unit at The University of Auckland where he developed a VPP for wing-sailed catamarans.

Richard G.J. Flay, BE(Hons), PhD, is Professor of Mechanical Engineering and Director of the Yacht Research Unit in the Department of Mechanical Engineering at the University of Auckland. He has had a longstanding research interest in the wind and sailing. His PhD degree was awarded for a study of wind structure based on full scale wind measurements. His Postdoctoral research as a National Research Council Visiting Fellow in Canada was focused on wind tunnel studies in a boundary layer wind tunnel. He then spent four years as an Aerodynamic Design Engineer in an Engineering Consultancy in Toronto where he worked on the design of several wind tunnels and environmental test facilities. Since 1984 he has worked at the University of Auckland, and in 1994 he designed the World’s first Twisted Flow Wind Tunnel.