SIMSPAR: AN EFFICIENT TOOL FOR MAST DESIGN AND TUNING

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SUMMARY

Which tool for mast design? Facing this question, mast designers have two main issues: empirical rules of thumb based on experience and Euler's formula or time-consuming industrial finite element programs. In this paper, Simspar, an efficient software dedicated to mast design and tuning, is presented. Starting from rig mechanics, the linear finite element method, based on a classical beam element model, is described. The mast tube stability problem equations are then derived, leading to buckling calculation. A method to improve consistency of mast deformation under compression is also proposed. The different steps of the mast design process are analysed: geometry meshing, sail load definition and mast tuning. A comparison with an industrial non-linear finite element package is carried out, showing the consistency of results. Influence of common hypothesis is then carried out. Finally some case studies are proposed.

NOMENCLATURE

This is required for all papers using a large number of symbols, abbreviations or acronyms.

- \( C \): Flexibility matrix
- \( C_d \): Drag coefficient
- \( E \): Young modulus
- \( \varepsilon \): Strain
- \( F_t \): Sail transversal force
- \( I \): Moment of inertia
- \( K \): System stiffness matrix
- \( L \): Element length
- \( H_e \): Sail centre of effort
- \( \rho_a \): Air density
- \( RM \): Yacht righting moment
- \( S \): Yacht sail area
- \( S_i \): Sail individual area
- \( \sigma \): Stress
- \( u, v, w \): Translation displacements
- \( \theta_x, \theta_y, \theta_z \): Rotation displacements
- \( x \): Node displacement vector

In section 2.1, the ~ superscript refers to symbols expressed into the global coordinate system.

1. INTRODUCTION

Mast and rig are essential parts in yacht building for various reasons. First of all, they carry the sails that are the engine of a yacht, and any rig failure can have dramatic consequences for the boat or its crew. Then, from a mechanical point of view, it is a complex and coupled system with a very high aspect ratio and important compression forces, and as such it is subject to stability problems. Loads applied on the mast are uncertain, difficult to measure, subject to huge variations depending on the sailing conditions and the sea state, so there is no simple way to evaluate accurately its behaviour. The mast and rig design have also a great influence on performances, firstly because of the height of centre of gravity that decreases yacht stability, then because its deflection under loads influences the flying shape of the sail and hence the sail efficiency. The mast designer has thus to deal with two opposite objectives: to obtain a lightweight structure, with a low centre of gravity, that can be efficiently controlled by the crew, and to obtain enough robustness to handle all the conditions that the yacht may encounter.

To face that challenge, mast designers have traditionally two main kinds of tools: Finite Element Method (FEM) or Euler's Formulas. The first method represents the state of art of structural engineering and provides very good results as long as loads are accurate. But the software licenses are quite expensive, and time-consuming to use because the user must input the whole geometry, loads and boundary conditions for each configuration or for any geometry change. Practically, these methods are only used to check the validity of a candidate design or to carry out a refined analysis of a design before building. The second method, the use of Euler's Formulas, that deals with mast stability, in combination with designer experience, appears to be a "rule of thumb" and leads to over-built rigs and overestimated safety coefficients to avoid any risk of failure due to the various approximations made along the design process. In this context, there is a clear need for a tool that provides accurate and faster results.

In this paper, we describe SimSpar, a software dedicated to mast design and tuning, based on a finite element method using beam elements. In section 2 we show the set of equations that are used to solve the finite element problem, leading to mast deflection and cable tension, then we derive equations used to evaluate mast stability under compression. In section 3, we go through the different steps of the mast design process showing how
the software has been adapted to fit the mast design problem. This include meshing issues to reflect quickly any geometry change, sail load calculation, mast tuning and how to deal with slack cables which is a problem with linear finite element method. In section 4, we carry out a comparison between linear and non-linear finite element method, using an industrial finite element package and we show the influence of an improved method to evaluate mast deflection. Finally some cases are proposed on various rigs that show some typical results obtained with SimSpar.

2. RIG MECHANICS

The mast and its rig are formed of sections (mast and spreaders) that can bear flexion due to their inertia as well as tension, compression and torsion, and of cables, called shrouds, that can only bear tension. Spreaders and shrouds provide lateral support to the mast. Spreaders are used to pull shrouds aside the mast and so to increase angles between those stays and the mast. Spreaders mainly encounter compression. Shrouds can only support tension and become slack under compression, upright shrouds are the verticals (or V’s) and angled ones are the diagonals (or D’s). Figure 1 shows a typical rig geometry including notation. Forces from the sails are directed to leeward, so a distinction is made between the windward side and the leeward side of the rig.

Figure 1: Rig geometry and notation

There are two main steps in SimSpar rig evaluation. The first one is to compute mast tube deformation and cable tensions using a finite-element method and the second one is to calculate mast stability and buckling coefficients under compression.

The model used into SimSpar to solve rig mechanics is a classical linear finite-element model using beam elements and is mainly based on the work presented by G.Dhatt and G.Touzot [1].

2.1 FINITE ELEMENT MODEL

Finite element methods are a very popular mean to solve structural problems. They are based on the subdivision of a wider problem into simpler problems evaluated at some points, so called nodes, deriving a relationship between nodal displacements $\mathbf{x}$ and nodal forces $\mathbf{F}$. The linear finite element method leads to:

$$\mathbf{K} \cdot \mathbf{x} = \mathbf{F} \quad (1)$$

where

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \cdots \mathbf{x}_n \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_1 \cdots \mathbf{F}_n \end{bmatrix} \quad (2)$$

and

$$\mathbf{F} = \begin{bmatrix} \tilde{F}_x \tilde{F}_y \tilde{F}_z \end{bmatrix} \quad (3)$$

So that knowing forces at nodes gives displacements at nodes.

2.1 (a) Coordinate system

The global coordinate system used in SimSpar is defined as follow:

- $z$ axis is upward along mast tube
- $y$ axis is parallel to water plane on starboard
- $x$ axis is perpendicular to $z$ and $y$ axis directed backward

The origin of the coordinate system is the mast collar for a keel-stepped mast or the mast step for a deck stepped mast.

2.1 (b) Beam and cable models

The beam model element used into SimSpar is based on the one proposed by Timoshenko which description can been found in [2] with six degrees of freedom per node (including torsion). This model is defined in element coordinate system and leads to a behaviour law between forces and displacements at node where displacements refer to node initial positions. Behaviour law is expressed in element local system coordinate system where $z$ axis is defined along main dimension of the beam. Nodal values for each node are thus three translations displacements along the $x$, $y$ and $z$ axis and three rotation variations along those axis, so called $u,v,w,\theta_x,\theta_y,\theta_z$.  

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The beam element can be defined as the superposition of equations (4) to (7). Those matrices are symmetric and lower coefficients have been omitted. Assembly of these four relations into a single matrix equation leads to equation (8), where $k$ is the element local stiffness and is a 12x12 matrix, $x$ is the node displacement vector and $f$ the element local forces, expressed into element local coordinate system:

$$k \cdot x = f \quad (8)$$

Cables only experience tensile stress, so only equation (4) has to be taken into account and the number of degrees of freedom is reduced to two per element.

$$\begin{bmatrix}
-N_i \\
N_j
\end{bmatrix} = \frac{E \cdot S}{L} \begin{bmatrix} 1 & -1 \\
-1 & 1 \end{bmatrix} \begin{bmatrix} w_i \\
w_j \end{bmatrix} \quad (4)$$

$$\begin{bmatrix}
-Tx_i \\
-My_j
\end{bmatrix} = \frac{E \cdot I_{xx}}{L^2 \cdot (1 + \phi_j)} \begin{bmatrix} 12 & -6L & -12 & -6L \\
(4 + \phi_j) L^2 & 6L & (2 - \phi_j) L^2 & u_j \\
12 & 6L & (4 + \phi_j) L^2 & \theta_{yj} \end{bmatrix} \quad (5)$$

$$\begin{bmatrix}
-Ty_i \\
-Mx_j
\end{bmatrix} = \frac{E \cdot I_{yy}}{L^2 \cdot (1 + \phi_j)} \begin{bmatrix} 12 & -6L & -12 & -6L \\
(4 + \phi_j) L^2 & 6L & (2 - \phi_j) L^2 & \theta_{yj} \\
12 & 6L & (4 + \phi_j) L^2 & \theta_{xj} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix}
-Mz_i \\
Mz_j
\end{bmatrix} = \frac{G \cdot J}{L} \begin{bmatrix} 1 & -1 \\
-1 & 1 \end{bmatrix} \begin{bmatrix} \theta_{zi} \\
\theta_{zj} \end{bmatrix} \quad (7)$$

2.1 (c) System Assembly

Cable properties are constant all along the system and external forces are located at its extremities so cables are modelled with a single two nodes element. On the other hand, mast tube is subdivided into several two nodes elements where section properties remain constant along each element. Then local stiffness matrix of each element is expressed into global coordinate system and assembled to lead to the complete system as expressed in equation (1). Solving this system, using a Gauss method, leads to node displacements. Then, going back to equation (8) provides local forces for each element.

2.1 (d) Cable pretension

The above theory is based on Hooke's law strain-stress relationship. In the axial direction this is expressed by:

$$\sigma_{zz} = E \cdot \varepsilon_{zz} \quad (9)$$

Equation (9) leads after integration on cable section to equation (4). This equation states that there is no strain in the structure if there is no deformation. This is also found through equation (1) where displacements are all zero if no force is applied on the structure. However, in the case of sailing yacht masts, we would like to consider the dock tune case as the initial case, taking into account cable pretension. One issue would be to give an initial displacement to chainplate node instead of the usual zero value but this is not acceptable for cable that are not linked to a chainplate such as V2. Instead, let's consider an internal stress in equation (9) leading to:

$$\sigma_{zz} = E \cdot \varepsilon_{zz} + \sigma_{zz0} \quad (10)$$

Integration of this equation on cable section gives:
\[ N = E \cdot s \cdot \varepsilon + s \cdot \sigma \]  

So equation (4) becomes

\[
\begin{bmatrix}
-N_i \\
N_j
\end{bmatrix} = \frac{E \cdot s}{L} \begin{bmatrix}
1 & -1 \\
-1 & 1
\end{bmatrix} \begin{bmatrix}
w_i \\
w_j
\end{bmatrix} + s \cdot \sigma \]  

(12)

The second term appears to be as an internal force in a cable of initial length L, such as the one produced by a turnbuckle and the internal strain can be calculated from the cable length reduction \( \delta \) by:

\[ \sigma = \frac{E \cdot \delta}{L} \]  

(13)

### 2.1 (e) Boundary conditions

In a structural finite element method such as the one used into SimSpar, boundary conditions appear as blocking some degrees of freedom at some nodes and then fixing displacement values for those equations. This is the case of cable chainplates that are fixed on the boat deck and as well for mast foot or mast collar (transversal and longitudinal displacement only, keeping vertical displacement free). Such boundary conditions must be removed from the system (1) as proposed by Dhatt [1] to lead to a new system of reduced dimension (actually those conditions are treated ahead of the stiffness matrix assembly to save computational time).

### 2.2 STABILITY AND BUCKLING

One of the major issues with the above method is that it cannot take into account moments due the displacement of axial force location. So this method is not able to evaluate mast stability and underestimate mast bending because only transversal forces are taken into account in the mast deflection calculation, however these issues are critical for mast scantling.

#### 2.2 (a) Buckling

Hence, mast stability is evaluated using the following method, proposed by Brunet [3] who derives the equation of a straight beam with variable inertia as set by Timoshenko [5]:

\[
(E \cdot I(z) \cdot x''')' + (N(z) \cdot x)'' = q(z)
\]  

(14)

where \( N(z) \) and \( q(z) \) are respectively the distribution of axial and transversal loads along the beam, to lead, after discretization of the problem, to the following matricial relation between forces and node displacements:

\[ \left[ C^{-1} + U \right] \cdot x = P \]  

(15)

where \( C \) is the flexibility matrix of the system, \( U \) is the vector of axial loads and \( P \) is the transversal load vector.

In his paper, Brunet derives the flexibility matrix from beam mechanical properties, but in our case, the stiffness matrix of the system has already been calculated, so the flexibility matrix can be evaluated using a unit dummy force method on each mast node.

Hence the critical loads \( U_c = \lambda \cdot U \) in the Euler sense (no transversal force) are such as:

\[
\left( C^{-1} + \lambda \cdot U \right) \cdot x = 0
\]  

where \( 1/\lambda \) are the eigenvalues of

\[ H = C \cdot U \]  

(17)

In SimSpar, buckling coefficients are evaluated twice, once for the longitudinal direction (along x axis) and once for the transversal direction (along y axis). In the case of a three dimensional analysis, this would lead to the first two buckling modes. In this way, the mast designer is able to analyse mast behaviour in both directions.

#### 2.2 (b) Mast deflection under compression

In the linear finite-element method as presented above, mast compression has very little influence on mast deformation, since forces are applied at node initial position, when the mast is a straight line. Actually there is coupling between mast bending and compression forces since those forces combined with mast deflection induce an extra bending moment all along the mast as shown in figure 3. So the linear finite-element method lead to an underestimated deflection but equations as derived by Brunet provide a mean to get a more accurate mast deflection curve solving equation (15). Improvements of this method, refer to as the flexion-compression deflection (FCD) method by us, compared to the finite-element deflection calculation, are discussed in section 4.

![Figure 3: Coupling between compression and flexion](image-url)
3. STEPS OF THE MAST DESIGN PROCESS

3.1 MESHING

The first step involved in the mast-design process is geometry meshing. Since cables have no transversal inertia, they only require two nodes. The mast tube needs to be subdivided into several two nodes elements. This line of nodes represents the neutral axis of the mast tube beam. The mast is firstly subdivided into panels (between spreader attachment, deck collar and mast foot) then panels are again subdivided into elements of similar length. Note here that the finite-element method has no specific requirement about element size but that the buckling method, based on the same mast tube subdivisions, but refined once more, requires elements to have a similar length. Cables are attached on mast neutral axis at existing nodes if any, or a new node is created but this last one will not be used for the purpose of buckling calculation. The user can control the number of element per panel or use the provided values. Meshing is completely automatic, depending on the previous parameters, hence the software immediately reflects any modification made by the user.

[Figure 4: Geometry and mesh]

Attaching cables on mast tube neutral axis instead of its real location on mast tube skin decreases the bending moment of the cable on the mast. This kind of theory has proven to be reliable but under some conditions, and especially for narrow masts having a high aspect ratio, it is an improvement to have those attachments at their real location on the mast tube. To model this, SimSpar inserts a new element between neutral axis and mast wall at node height as shown in figure 5.

[Figure 5: cable attachment]

3.2 SAIL LOADS

One of the major issues when dealing with mast design is the ability of the mast designer to define critical loads applied on the structure. It is well known that for a sailing yacht rig the following conditions: closed hauled, broach under spinnaker and running under spinnaker belong to those critical sailing conditions that a robust mast must be able to face\(^1\). So, SimSpar provides sail load calculation for those cases, although the user has the possibility to input his own loads for any conditions.

3.2 (a) Close-hauled

The first step is to evaluate transversal force on each sail and its centre of effort. The sum of each sail moment has to balance the righting moment of the yacht depending on sailing conditions:

\[ RM = \sum_i Ft_i (He_i - H_k) \]  

We assume all sails to have the same lift coefficient, hence transversal force for each sail is the ratio of individual sail area over the total sail area:

\[ Ft_i = \frac{S_i}{S} \cdot Ft \]  

Combining equations (18) and (19) leads to transverse force for each sail:

\[ Ft_i = \frac{S_i \cdot RM}{\sum_i S_i(He_i - H_k)} \]  

3.2 (b) Broached

In that condition, we assume that the mainsail is fully eased so that transversal force is only coming from spinnaker. Moment of spinnaker force have to balance the righting moment of the boat for this condition.

\[ RM = Ft \cdot (He_k - H_k) \]  

Hence transversal force for spinnaker is deducted from the above equation.

3.2 (c) Downwind

In that case, righting moment cannot be used any more since yacht is sailing upright or close to be, therefore we have to deal with the apparent wind speed. Sail is assumed to develop drag only and no lift. Drag coefficient is set to 1.1. Hence thrust of the spinnaker is:

\[ Ft = \frac{S_i \cdot RM}{\sum_i S_i(He_i - H_k)} \]  

1 Of course, those cases are not thorough and a careful examination of all critical loads must be carried out for each design.
From our experience, where the crew seeks for performance than on a cruising can expect to have much more tension on a racing boat on the sheet tension and thus of the boat program. We forces in the sail, and that the leech tension will depend highly depend on the leech tension through membrane value for this coefficient since the halyard tension will.

One must note that it is quite difficult to get an accurate calculation. So the user must be very careful and shall avoid any slack cable in the model. Such a cable must be

\[ F = \frac{1}{2} \rho_a \cdot V_a^2 \cdot S \cdot C_d \]  

(22)

3.2 (d) Sail load distribution

Once we know the transverse force on a sail, we have to distribute the loads at sail corner with respect to the sail balance. As a sail has quite a high aspect ratio we assume that the clew and tack points are located at the same height. Considering the height of the head, the side force at this point and the side force at clew and tack points, the balance of the sail leads to:

\[ Ft = F_h + F_{cl} \]

\[ Ft \cdot H = F_h \cdot h_h + F_{cl} \cdot h_{cl} \]  

(23)

leading to:

\[ F_h = \frac{F_t \cdot H - h_{cl}}{h_h - h_{cl}} \]  

(24)

This load is then applied to the node carrying the headsail stay or distributed on small portion of the mast at headsail location in the case of a mainsail.

Loads on the mast from the mainsail stem not only from transverse force but also from leech tension that induces a longitudinal load that cannot be neglected. This load highly depends on the mainsail twist, adjusted while sailing through the mainsheet tension and traveller, and also on the sail roach and on sail cloth. Carbon or Kevlar reinforced sail cloth are much stiffer than polyester sail cloth, leading to flatter sails that induce more sheet tension and bigger load on the mast. To model those effects, SimSpar provides a mainsail sheet adjustment that range from cruising with a twisted and low aspect ratio mainsail to racing with a flat mainsail with roach. According to our experience this load is mainly concentrated at head sail, hence it is distributed on a small portion of the mast in SimSpar model.

3.2 (e) Halyard Compression

Halyard compression is one of the main source of compression encountered in the mast tube, so they have a great influence on mast stability and buckling coefficients. For each sail, the halyard tension is defined as a function of sail transverse force:

\[ T_h = k_h \cdot F_t \]  

(25)

One must note that it is quite difficult to get an accurate value for this coefficient since the halyard tension will highly depend on the leech tension through membrane forces in the sail, and that the leech tension will depend on the sheet tension and thus of the boat program. We can expect to have much more tension on a racing boat where the crew seeks for performance than on a cruising boat. From our experience, \( k_h \) ranges from 1.5 to 3. A simplified method for the calculation of this coefficient can be found in [4].

3.3 MAST TUNING

SimSpar provides two different ways for mast tuning. The first one acts as a turnbuckle as shown in paragraph 2.1 (e) and can be applied on any cable belonging to the structure. Nevertheless this method cannot be used for mast foot and deck collar, so the user can set the initial displacement for those nodes. The mast foot can be moved forward and aft as well as the deck collar. This adjustment has a great influence on mast deflection and the main parameter is the difference of displacement between those two points. The last tuning mean is the possibility to move the mast foot up and down in order to simulate a mast jack cylinder, which is an usual method on a large number of yachts.

Leeward shrouds are relaxed under loads and would become slack if no pretension is applied on them. That leads to an significant loss of mast stability, that could go up to mast buckling and failure, especially if cap shrouds (outermost lateral shrouds) become slack. On the other hand, a large pretension induces a lot of compression in the mast due to shroud angle and would also lead to a loss of stability of the structure.

On racing yacht, mast tuning is hugely used to adjust the mainsail luff curve (and in turn the sail profiles) and the forestay sag. Those effects are often difficult to apprehend due to the interaction between all the components of the rig structure, but since SimSpar shows mast deflection both in the longitudinal and lateral direction, it provides a powerful mean to explore the influence of mast tuning. SimSpar also provides an estimation of forestay sag, based on forestay tension and headsail sheet tension. Forestay sag is given both for longitudinal and lateral directions, the former value influences headsail profile camber whereas the second one induces a rotation (or twist) of headsail sections.

3.4 SLACK CABLES

The major issue with slack cables when using a linear finite element model is that cables under compression push on the structure, leading to an inconsistent calculation. So the user must be very careful and shall avoid any slack cable in the model. Such a cable must be
removed from the calculation, then the stiffness matrix rebuild, or its Young modulus sets to zero (but this could lead to a singular stiffness matrix). This task can be quite long and boring, but it quite easy to find which cables are under compression looking at their tension. The opposite issue - should that cable, defined as slack and so having been removed from the calculation, be under tension? - is much more tricky because no information is available about that element.

To get rid of that problem, this element, assumed to be under compression, is kept in the model but its Young modulus is set to a very small value, in order not to change the calculation result if the cable is actually under compression and to be able to check if the cable should be in tension.

If the state of a cable is found to be wrong, it is toggled and the calculation is run again until all cables are in the correct state. This method has proven to be efficient and to converge easily, however it has not been shown that it shall converge.

3.5 SOFTWARE

As a tool dedicated for practical mast design, SimSpar bears a very special attention to ergonomics and efficiency. A set of various rig layout is provided in order to start quickly with a geometry close to the final one, a library of mast section and cable properties is included, which can be extended by the user. The user interface offers interactive design since calculations are more or less instantaneous. Mast design specific tools are provided such as mast deflection, mast camber charts and buckling deflection that allows user to visualize mast weakest part.

It takes only a few hours to make a study using SimSpar. Having geometry and material defined can be achieved within half an hour and it takes only a few minutes more to get the first calculation done using SimSpar's default loads. A full rig evaluation can be carried out within half a day or one day depending on the project complexity. Using an industrial and all purpose finite-element package would take about one day to have a single case calculated and each extra cases would take about half a day, so a complete rig evaluation would take several days without testing various rig configurations.

4. COMPARISON WITH NASTRAN

In this section we present two comparisons made against the commercially available finite element package Nastran that have nonlinear capabilities and is a multi-purpose FEM whose reliability is proven but not dedicated to any specific calculation. The main differences between the two programs are: a) since Nastran is not dedicated to mast calculation, the meshing process takes a longer time than with SimSpar, and b) Nastran user has to define boundary conditions and to compute and input loads from the sails himself. The first comparison is made using Nastran only with both linear and non-linear theory. The second comparison shows differences between SimSpar and Nastran calculation and also shows results of mast deflection calculation using the method proposed in section 2.2.

4.1 LINEAR VS NON-LINEAR

One of the main issues when using a linear method is to know how accurate it is compared with a non-linear method. Here, we have compared the linear method result with non-linear incremental method results using Nastran. The rig is a two spreader mast. The two following figures show mast longitudinal and lateral deflections. Non-linear calculation lead to a more important deflection in the longitudinal direction whereas lateral deflections are more or less the same in both calculations. In most of the case that the authors have experienced, non-linear calculations usually lead to more mast deflection in both directions than linear calculation.

<table>
<thead>
<tr>
<th></th>
<th>Linear [t]</th>
<th>Non-linear [t]</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1 leeward</td>
<td>0.51</td>
<td>0.49</td>
<td>-3.2 %</td>
</tr>
<tr>
<td>V1 windward</td>
<td>1.74</td>
<td>1.75</td>
<td>+0.3 %</td>
</tr>
<tr>
<td>D1 leeward</td>
<td>0.51</td>
<td>0.54</td>
<td>+6.0 %</td>
</tr>
<tr>
<td>D1 windward</td>
<td>1.71</td>
<td>1.72</td>
<td>+0.9 %</td>
</tr>
<tr>
<td>V2 leeward</td>
<td>0.40</td>
<td>0.36</td>
<td>-9.2 %</td>
</tr>
<tr>
<td>V2 windward</td>
<td>1.00</td>
<td>0.99</td>
<td>-0.9 %</td>
</tr>
<tr>
<td>D2 leeward</td>
<td>0.11</td>
<td>0.13</td>
<td>+20.6 %</td>
</tr>
<tr>
<td>D2 windward</td>
<td>0.74</td>
<td>0.75</td>
<td>+2.3 %</td>
</tr>
<tr>
<td>D3 leeward</td>
<td>0.40</td>
<td>0.37</td>
<td>-9.2 %</td>
</tr>
<tr>
<td>D3 windward</td>
<td>1.01</td>
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<td>-0.8 %</td>
</tr>
<tr>
<td>Forestay</td>
<td>3.18</td>
<td>3.18</td>
<td>0.0 %</td>
</tr>
<tr>
<td>Backstay</td>
<td>1.85</td>
<td>1.83</td>
<td>-1.0 %</td>
</tr>
</tbody>
</table>

Table 1: Cable tension using linear and non-linear models
The above table shows cable tension in tons and variations between both calculations. Results are very similar, all variations are within 10% but for the leeward D2 with a 20.6% variation (which is calculated over a very low tension, so leading to huge relative variation whereas absolute variation is in the same order of magnitude than other variations shown in this case). V's tension show very little variation with values within 4%. On the windward cables, that encounter much more tension than the leeward ones, the maximum variation is only 2.3%.

So it can be seen through this example that the main difference between linear and non-linear methods occurs on mast deflection and that cable tensions are pretty well calculated by the linear method adopted by SimSpar.

4.2 MAST DEFLECTION

In this section, a comparison is made between SimSpar and Nastran on a one swept spreader mast, including two forestay and a backstay, in order to evaluate the effects of the different theory on mast deflection calculation. Mast layout under Nastran is shown on figure 7.

Mast deflection in longitudinal and transversal directions are shown for four different calculations: Nastran linear model, SimSpar linear model, Nastran non-linear model, and SimSpar FCD method.

The first point is that there is very little difference between the two linear calculations, which is consistent since we have reproduced the same model using two different tools, this also proves that input are consistent in both programs. Then the non-linear calculation shows much more deflection than the linear one, especially regarding transversal deflection where non-linear displacements are twice as much as the linear case. The other point is that SimSpar results using the FCD method are very close to Nastran non-linear results. This proves that the main effect taken into account in the non-linear method are the second order moment induced by the displacement of compression forces with deflection, and this effect is correctly reproduced by the proposed method in SimSpar.

![Figure 7: Mast layout](image)

Figure 7: Mast layout

Mast deflection in longitudinal and transversal directions are shown for four different calculations: Nastran linear model, SimSpar linear model, Nastran non-linear model, and SimSpar FCD method.

4.3 BUCKLING CALCULATION

Here we show a comparison of stability calculation results between SimSpar and Nastran. Those calculations provide the so called buckling coefficients that are the inverse of the factor that one should apply to the current external loads to lead to a stability failure of the mast and are calculated as eigenvalues of matrix $H$ as expressed in equation (17). A value of 1 for those buckling coefficients means that the mast is subject to buckling and a value lower than 1 refers to a safer configuration. For cruising yachts, buckling coefficients are likely to be lower than 0.6 to ensure a good security margin and for racing yachts, buckling coefficients can go up to 0.7 or 0.75. One difference between the two software is that Nastran does not take into account cable pretensions into the buckling coefficient evaluation, because pretensions are input as thermal dilatation (and the material dilatation induces an internal stress into the cable) and are not considered as external to the structure. In SimSpar, we have included axial loads on the mast coming from shroud pretension in the axial load vector $U$ in equations (15) through (17) so that pretension is taken into account in buckling coefficient calculation. Another difference is that Nastran carries out a three-dimensional stability calculation whereas SimSpar computes twice a two-dimensional problem. The advantage of the later method is to provide separated information in both longitudinal and lateral directions, hence the mast designer is able to analyze it. The two coefficients calculated by SimSpar correspond to the two first buckling modes calculated by Nastran, and the weakest direction is given by the first mode. Actually, mast stability is a three-dimensional problem but it appears that the first buckling mode,
which is the critical one, is only slightly affected by three-dimensional effects whereas the second one is much more influenced. The following table shows buckling coefficients for the mast used in section 4.1.

<table>
<thead>
<tr>
<th></th>
<th>Longitudinal</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>SimSpar</td>
<td>0.484</td>
<td>0.371</td>
</tr>
<tr>
<td>Nastran</td>
<td>0.390</td>
<td>0.322</td>
</tr>
</tbody>
</table>

Table 2: Buckling coefficient

We notice that Nastran coefficients are much lower (about 20%) than the ones calculated with SimSpar. This stems from the fact that pretensions are not taken into account in Nastran, as explained above. A calculation using the same geometry but removing cable pretension (which is unrealistic) leads to a 0.3% difference between coefficients from the two software. On the other hand, since buckling deflection are the same with both software, this proves again that buckling is calculated in the same way and that the critical force in Euler sense is similar, but that it is the reference compression used to calculate buckling coefficients that differs. Our point of view is that mast compression due to shroud pretension is a significant part of the total mast compression and therefore has to be taken into account in buckling coefficient calculation.

5 CASE STUDY

5.1 PRETENSION AND STABILITY EFFECTS

In this case, we show how shroud pretension can affect mast stability. The yacht is a cruiser-racer and the mast has two swept spreaders, a backstay and no runner. Mast layout is shown in figure 9 below.

Figure 9: Cruiser-racer mast layout

In the first case, the mast has been tune in a satisfactory manner but leeward shrouds are slack except the D1 that has a very low tension (around 10 kg), so that the cable is practically slack but still existing in the system. Buckling coefficients are 0.48 in the longitudinal direction and 0.46 in the transverse one. Then, releasing one single millimetre on D1 turnbuckles leads to a longitudinal buckling coefficient of 0.78 and to a transversal value of 0.49. The leeward D1 is now slack as all other leeward cables. Clearly, such a buckling coefficient is not acceptable for a cruising yacht and mast designers or naval architects could wonder why their design has switched from a safe state to a doubtful one. What happens here is that the leeward D1 stiffness is removed from the stiffness matrix when it becomes slack, leading to a much larger buckling coefficient, however this cable is likely to be tightened again under a small variation of mast deflection, which can quickly occurs if the mast has stability problem and so is subject to large displacements. Once this cable is back under tension, mast stability is recovered, but the mast is likely to pump fore and aft. SimSpar provides then a way to anticipate this situation and also a mean to set shroud pretension specification in order to avoid this problem.

5.2 MAST TUNING

Competitors know that tuning the mast correctly is a necessary condition to achieve a good boat speed while racing. Nevertheless this a difficult task and often a long time process that require a lot of experience of sailing and a good knowledge of the rig you are sailing with. The following mast is set on a racing yacht, it is a two swept spreader mast with a single backstay. Since it has no runner, the forestay tension cannot be easily adjusted and the crew has to deal with rig set up, mainsail sheet tension and backstay.

Figure 10: Racing yacht mast layout

In figure 11, we show variations ratio of mast camber for various adjustments of standing and running rigging with respect to the camber of a reference case. Mast camber is taken as mast deflection relative to mast chord line, taken between mast foot and head. So a value greater than 1 means that mast has more camber than the reference case. This reflects how the mainsail shape would be affected by those adjustments, more camber leading to a flatter sail shape. Total mast height is 15m, mast head is on the
The order of magnitude of standing rigging adjustments are realistic for that mast but backstay variation has been kept to a low value for the purpose of comparison.

![Figure 11: Mast camber variations](image)

Backstay and forestay have nearly the same effect at the top of the mast but their influence is different at the bottom of the mast. This also indicates that if the backstay is tightened to its maximum due to hardware limitation, it can be interesting to have more pretension on the forestay to add up these two effects. Mast foot position has a very significant influence on the bottom part of the mast and clearly appears as the best mean to control camber in that area. Actually a deck collar displacement would have the same effect with mast foot fixed and the important figure is the difference of displacement between these two nodes. Cap shrouds and diagonals appear to have influence in middle part of the mast. Table 3 below shows forestay sag values for the same set of adjustments.

![Figure 12: M1 layout](image)

Maximum variation is found for D2 and RD3 to be 20% but calculated over low value, so that a small difference lead to an important variation. On the other hand we had also some differences between both side pretension at dock up to 0.4 T, that we assumed to come from mast windage. Variations for other shrouds are between 5% and 13%, hence we can say that SimSpar calculation is in good agreement with measurement.

<table>
<thead>
<tr>
<th>Shroud</th>
<th>SimSpar (T)</th>
<th>Measure (T)</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>13.7</td>
<td>12.6</td>
<td>8.8%</td>
</tr>
<tr>
<td>D1</td>
<td>5.7</td>
<td>5.1</td>
<td>11.8%</td>
</tr>
<tr>
<td>V2</td>
<td>10.9</td>
<td>9.5</td>
<td>12.8%</td>
</tr>
<tr>
<td>D2</td>
<td>2.9</td>
<td>2.3</td>
<td>20.0%</td>
</tr>
<tr>
<td>V3</td>
<td>10.9</td>
<td>10.4</td>
<td>5.1%</td>
</tr>
<tr>
<td>RD3(^3)</td>
<td>0.5</td>
<td>0.6</td>
<td>20.0%</td>
</tr>
</tbody>
</table>

Table 4: Difference between windward and leeward shroud tension

The fact is that using this mast, the crew had many difficulties to tune it the way they wished and especially they could not control efficiently the top part of the mast. Then, we have used the M2 mast, which layout is shown on figure 13, that has proved to be much easier to tune than M1 and gave better performance.

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2 International America's Cup Class

3 RD : Reverse diagonal
Both masts have been modelled with SimSpar and tuned at the best of user's skill. The following figures show longitudinal and lateral deflections for both masts in similar conditions. Obviously, curves are really different. M1 lateral deflection is much more important, with a maximum bend (maximum deflection with respect to chord length over mast height) of 0.48%, than M2's one which is 0.05% and whereas M2 deflection is close to be a straight line, M1 bend has a "S" shape in the middle part of the mast. Looking at longitudinal deflection, M2 shows a fair curve with smooth curvature and a moderate bend of 0.50% whereas M1 has significant curvature variation above mast mid-height and much more bending, 1.07%, than M2. From a sail designer point of view, using the same sail on both masts leads to a dramatically different sail shape, and the luff curve would have to be adapted for each mast.

Another interesting point with this kind of mast, having a aspect ratio (mast total height over rig width) around 10.3 which is a very high value compared to common rigs that are more likely to have an aspect ratio around 6, is the influence of cable attachment location as shown in section 3.1. In the case of IACC rigs, moment due to shift of cable attachments from mast neutral axis is relatively more significant than the one found on wider rig. Actually this effect is not really significant for low aspect ratio mast whenever one can note some differences. The following table shows the effect of this model for the IACC masts as presented here:

<table>
<thead>
<tr>
<th>Cable</th>
<th>Pretension [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>-65</td>
</tr>
<tr>
<td>RD1</td>
<td>33</td>
</tr>
<tr>
<td>D2</td>
<td>-42</td>
</tr>
<tr>
<td>RD2</td>
<td>22</td>
</tr>
<tr>
<td>D3</td>
<td>-27</td>
</tr>
<tr>
<td>RD3</td>
<td>10</td>
</tr>
<tr>
<td>Mastjack</td>
<td>55</td>
</tr>
</tbody>
</table>

Table 5: Effect of shroud attachment location

Figure 15 shows slack shrouds for mast M2 under load, slack shrouds are greyed and shrouds under tension are black. Notice that windward diagonals are under tension whereas leeward diagonals are slack and that leeward reverse diagonals only are under tension. To get that figure both diagonals and reverse diagonals have to be slack at dock (no sail load). Table in figure 15 provides turnbuckle adjustments for this rig, a positive value indicates a pretension and a negative value means that the turnbuckle has been eased.
The most significant effect occurs on lateral buckling coefficients that decrease dramatically when using the enhanced model with shroud attachment located on mast wall. In both cases, calculation with the basic method leads to a potentially unsafe mast whereas stability issue looks much more comfortable in the second case. This stems from the fact that the extra moment due to the location of attachment improves lateral mast stiffness. Mast deflection is also affected and the effect is once again more sensitive in the transversal direction than in the longitudinal one, a logical output since spreaders are in the transversal plane.

**CONCLUSIONS**

We have described the numerical method adopted into SimSpar and highlighted the enhancements made to improve the basic linear theory. We have also shown the reliability of this method and proved that it is highly adapted to the evaluation of sailing yacht mast behaviour. Main effects that occur in rig mechanics are taken into account and SimSpar focuses on getting quickly a full evaluation of a rig layout under various sailing conditions. It has been shown that the main non-linear effect is due to displacement of force application point, leading to extra stress into the mast that are not taken into account by the linear theory, it has also been shown that this phenomenon has a great influence on mast deflection and little effect on shroud tension. A method has been proposed to improve mast deflection calculation as provided by the linear method that mostly fill the gap with non-linear calculation. Nevertheless, one should keep in mind that some effects are not taken into account such as sail stiffness. Indeed, sail forces act only as a load on the structure, whereas it should also be included in the stiffness matrix, leading to an overestimation of buckling coefficients, especially in the longitudinal direction. Those effects, that are assumed to be small, are quite complicated to evaluate and may be not neglected in some cases such as dinghy mast where section inertias are small. Another effect is hull elasticity, which is deformed under rig loads, nevertheless this effect may mainly change turnbuckle set up required to reach a given rig tension. Through last section examples, it has been shown how SimSpar is able to deal with common mast problems. This software is based on a robust theoretical background and provides efficient tools for mast design and tuning and the authors are convinced that it will be a useful tool for all people dealing with mast design.

**REFERENCES**