DEVELOPMENT OF A THREE-DIMENSIONAL INVERSE SAIL DESIGN METHOD

Julien Pilate¹, julien_pilate@hotmail.com
Frederik C. Gerhardt², fger003@aucklanduni.ac.nz
Richard G. J. Flay³, r.flay@auckland.ac.nz

Abstract. A code which generates the camber shape of a sail from a desired sail plan-form, sail twist distribution and surface pressure map has been written. This is an iterative 3D inverse sail design code. The method initially uses inverse thin aerofoil theory, applies this to the desired pressure map and creates an initial sail shape. A theory which gives a relationship between the change in the pressure map and the change in the sail camber was developed and is described. The code applies that theory to the difference between the desired pressure map and the pressure map of the initial sail shape. The calculated camber difference is added to the initial shape to give an improved shape with a pressure distribution closer to the desired one. This process is repeated until the generated sail produces the desired pressure map. Validation tests were performed by generating a pressure distribution from a known sail shape using a VLM code, and then the method described in the paper was used to find the shape from the pressure distribution. The sail shape was successfully obtained in as few as five iterations, with a maximum error of only about 0.2% of the sail chord, which is acceptable in sail design practice.

NOMENCLATURE

\begin{itemize}
\item $a_n$  Fourier coefficients of the cambered aerofoil
\item $C_p$  Pressure coefficient
\item $C$  Chord length
\item $dC_p$  Difference between $C_{p_{\text{low}}}$ and $C_{p_{\text{up}}}$
\item $\text{LE}$  Leading Edge
\item $\text{TE}$  Trailing Edge
\item $u$  Velocity on the cambered aerofoil
\item $U_\infty$  Velocity of the free stream
\item $\text{VLM}$  Vortex lattice method
\item $X$  Chordwise coordinate
\item $Z$  Camber coordinate
\item $\alpha$  Angle of attack
\item $a_{\text{twist}}$  Geometric sail twist
\item $\beta_{\text{AW}}$  Apparent wind angle
\item $\gamma$  The strength of a vortex sheet
\item $\theta$  Angular coordinate
\item $\Gamma$  Circulation
\item $\text{RANSE}$  Reynolds-averaged Navier-Stokes Equations
\end{itemize}

1. INTRODUCTION

Many sail designers apply their collected experience to shaping the sail. Analysis tools and their experience help them choose one design over another, but a methodology to achieve an aerodynamically optimal solution is usually unavailable.

The process of inverse sail design is currently being investigated at the Yacht Research Unit of the University of Auckland. In this research, we have aimed to develop a global process which results in the design of the “best” sail under constraints (true wind speed, true wind angle, heel, etc). The project has been split into two parts: the span-wise (along the mast) problem, and the chord-wise problem. The final goal is to link the codes of both parts.

The first, span-wise part of the problem was investigated by Junge [1]. There the optimal span-wise distribution of lift was found using a numerical optimisation technique. Once an optimum lift distribution has been found it must be decided how to obtain an actual 3D sail shape which will produce the desired aerodynamic characteristics when used on the water. This is the chord-wise problem. In this paper we focus on the chord-wise part of the inverse sail design process.

Some investigations of the inverse process of the chord-wise problem have been made previously [2,3,4,5]. In those investigations, the inverse method was used such that the sectional pressure distribution was specified and the foil shape was derived from this. After discussion with Fallow and Sergent (Sail Designers at North Sails (NZ) Ltd), the decision was made to investigate a way to generate a sail using a 3D pressure map. Such a tool could be used by sail designers to explore new flying shapes.

2. THIN AEROFOIL THEORY

Thin aerofoil theory constitutes one way of calculating the flow around an aerofoil. This theory makes the assumption that the camber is small in relation to the chord. Thin aerofoil theory is based on the following observations:

- By placing vortices in the flow, lift can be generated independently of the shape of the aerofoil from where the circulation derives.
- The angle of attack and the asymmetry of the profile are the effects which produce the lift.

In thin aerofoil theory the aerofoil-bound circulation is represented by a continuous sheet of vortices. The distribution of the vortices in this sheet must be defined.
in such way that the flow leaves the trailing edge smoothly (Kutta Condition). To simulate the incidence, the aerofoil is replaced by a flat plate at a certain angle of attack. The final flow is the superposition of the solution for the camber problem and the incidence problem. Both of them generate lift.

To apply this 2D theory to a sail section, its thickness is assumed to be infinitely small and the sail is “cut” into a number of aerofoil-like slices. By treating each section like a thin aerofoil, the sectional pressure distributions can be calculated.

The expression for the pressure distribution over a cambered thin aerofoil is presented below based on the equations of thin aerofoil theory as summarised in [6]. Figure 1 illustrates the fundamental idea of thin aerofoil theory. For a slightly cambered foil, a vortex sheet of strength $\gamma(X)$ can be placed on the chord rather than on the camber line. The bound circulation $\Gamma$ is then calculated by integrating the strength of the vortex sheet along the chord.

$$\frac{Z}{X} = \frac{1}{2}(1 + \cos \theta) \cdot \frac{\alpha}{\alpha}$$

Equation (1) allows the velocity distribution over a thin cambered aerofoil to be computed.

If the camber line is expressed using a polynomial approximation the shape of the camber line can be transformed from Cartesian to Polar coordinates with the aid of Equation (3). Note that it is essential that the theory is developed in a coordinate system where the thicknesses at the leading and trailing edge, $Z(LE)$ and $Z(TE)$, are both zero.

Figure 2 illustrates the principle of superposition. Two flows are superimposed to get the desired solution.

### 3. INVERSE THIN AEROFOIL THEORY

#### 3.1 General approach

The previous section focused on the analysis problem i.e. a sail shape is specified and the resulting pressure distribution is desired.

In this section the aerodynamic characteristics are specified, and the shape required to obtain them is computed. This is the inverse approach.

In equation (1) applied to the inverse approach, the angle of attack $\alpha$ and $\Delta Cp$ are known and the camber is to be found. For this, the Fourier Coefficients $a_n$ that generate those characteristics are required in order to find the shape.

To tackle this problem, the degree ("$n$") of the system that will set the number of terms to describe the cambered aerofoil and the equation of the system must firstly be defined. Thus there are "$n$" unknown Fourier coefficients. To close the problem, equation (1) is expressed "$n$" times. Practically, "$n$" points on the $\Delta Cp$ curve are selected and solved in a system of $n$ equations. The result is a matrix containing the Fourier coefficients which describe the aerofoil.
3.2 Constraints

In addition to making the camber line a streamline, it has to be ensured that the camber is zero at the leading and trailing edges. To do so, an additional constraint has to be added to the system of the Fourier coefficients, and thus an over-determined system is created.

3.3 Example

In the following example, we take \( n = 4 \), which means that four \( \Delta C_p \) values at four different positions \( \theta \) along the curve are used. A required angle of attack has to be specified as well. From (1) the following system of equations is then obtained:

\[
\begin{bmatrix}
\Delta C_p - 4\sin \theta \\
\Delta C_p - 4\sin \theta \\
\Delta C_p - 4\sin \theta \\
\Delta C_p - 4\sin \theta \\
\end{bmatrix} = \begin{bmatrix}
4^2 \cos(1 \theta) \\
4^2 \cos(2 \theta) \\
4^2 \cos(3 \theta) \\
4^2 \cos(4 \theta) \\
\end{bmatrix} \sin(\theta)
\]

In the inverse problem \( \Delta C_p \) and \( \alpha \) are known and the Fourier coefficients \( a_n \) are unknown.

As proven in [7], the additional constraint for \( n = 4 \) is:

\[
a_1 + a_3 = 0 \quad (4)
\]

The end result is the following over-determined system:

\[
\begin{bmatrix}
\Delta C_p - 4\sin \theta \\
\Delta C_p - 4\sin \theta \\
\Delta C_p - 4\sin \theta \\
\Delta C_p - 4\sin \theta \\
\end{bmatrix} = \begin{bmatrix}
4^2 \cos(1 \theta) \\
4^2 \cos(2 \theta) \\
4^2 \cos(3 \theta) \\
4^2 \cos(4 \theta) \\
\end{bmatrix} \sin(\theta)
\]

The terms \( a_n \) can now be deduced by using a least squares approach. These Fourier coefficients give the closest pressure distribution to the desired \( \Delta C_p \) curve, whilst also satisfying the thin aerofoil equation (1).

Figure 3 shows the desired pressure (solid line) and the result of the inverse thin aerofoil theory (dashed line). Following the principle of superposition, the result is the sum of the \( \Delta C_p \) due to the cambered aerofoil at zero angle of attack and the \( \Delta C_p \) due to the flat plate.

The interesting point of the inverse thin aerofoil theory used herein is that we are dealing with an analytical solution: there is a continuous vortex distribution along the chord. This means that there is only one solution of the inverse problem so the shape which generates the required pressure distribution is found directly. In other inverse methods [2,3,4,5], the assumption of small camber is not made and the vortices are placed on the camber. In an inverse method, where the shape of the camber line is unknown, this complicates matters. Thus one has to use an iterative approach where the geometry is modified in a systematic manner until the specified pressure is obtained.

4. INVERSE PROCESS IN 3D

The previous section describes a way to find a camber by specifying the pressure distribution. The goal of the following section is to find an inverse process for a real sail, which is a three-dimensional problem. This paper focuses on a single sail.

4.1 Generation of a sail shape using a 2D theory

To apply inverse thin aerofoil theory to a 3D problem, different “cuts” of the pressure map along the mast are considered and then inverse thin aerofoil theory is applied to each of those to generate a camber. Note that inverse thin aerofoil theory is two-dimensional in nature, which means that this theory cannot account for the vertical flow component due to tip vortices and heel. Thus some problems where the 3D effects are strong, e.g. at the head and the foot of the sail, can be expected. To avoid this problem inverse thin aerofoil theory has been applied only to the central 80% of the sail and interpolation has been used at the head and the foot.

To ensure that a realistic pressure map is used for testing purposes, the pressure distribution of an existing sail has been used. The mainsail of the legendary New Zealand Half-tonner: “Waverider” was chosen for this purpose. The flying shape was found using a finite element analysis (using Flow ® /Membrain ® from North Sails) assuming a rigid curved mast. The resulting flying shape was run in a Vortex Lattice Method program (VLM) to generate its pressure map. This type of code solves the Laplace equation and can predict an accurate (but not exact) pressure map for upwind sails. The calculated pressure was then used as input into the 3D inverse method. At the end of the process, the aim is to compare the original flying shape and the shape from the code and to quantify the accuracy of the inverse method.

The VLM used was “Sail++” developed by Werner [8]. “Sail++” calculates the pressure difference (\( \Delta C_p \)) across the sail. The input parameters for the VLM were:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{AW} )</td>
<td>5.0 [m/s]</td>
</tr>
<tr>
<td>( \beta_{AW} )</td>
<td>22 [deg]</td>
</tr>
<tr>
<td>Heel</td>
<td>10 [deg]</td>
</tr>
</tbody>
</table>

Figure 3: Inverse thin aerofoil theory
A semi-cosine spacing of the panels was chosen and the influence of mast and rigging on the flow was neglected. To apply inverse thin aerofoil theory the angle of attack of each section “i” along the mast was calculated using the apparent wind angle and the local twist:

$$\alpha_i = \beta_{AW} - \alpha_{Twist}$$  

Thus the luff and the leech have to be specified or, in other words, the sail plan-form and twist need to be known.

The sectional solution of the inverse thin aerofoil theory is called “Sail Start”.

The Figure 4, 5 & 6 show results obtained by using thin aerofoil theory to determine the sectional mainsail shape at ¼, ½ and ¾ span respectively. Note that the results are non-dimensional. A significant difference between the original sail and the sail that the inverse thin aerofoil theory creates can be observed. This difference is caused by three-dimensional effects neglected in thin aerofoil theory.

It is interesting to note that the camber of “Sail Start” is lower than the original one. Indeed, in 3D we should have more camber to develop the same sectional pressure distribution compared with 2D. The reason is that the vertical flow decreases the pressure difference in the chord-wise direction.

The “Sail Start” is different from the original one and, of course, will generate a different pressure distribution. The pressure distributions at the different span-wise locations are illustrated in Figure 7, 8 & 9.
A way to modify “Sail Start” which is a function of the difference between the two pressure distributions thus has to be found.

Note that the VLM does not predict a $\Delta C_{p}$ of zero at the TE. This is caused by the numerical discretisation of the VLM code.

4.2 Extension of the Thin aerofoil theory

To modify “Sail Start” an extension of the inverse thin aerofoil is used. Figure 10 shows the desired pressure map, and the “Sail Start” pressure map, and illustrates the basic idea of extended thin aerofoil theory.

The difference in $\Delta C_{p}$ is obtained by subtracting one pressure map from the other and then applying the inverse thin aerofoil equation (1),

$$
delta \Delta C_{p} = \Delta C_{p, \text{from 2d}} - \Delta C_{p, \text{original}}
$$

Thus,

$$
delta \Delta C_{p} = 4 \cos(\alpha) \left( \frac{\alpha}{\sin(\alpha)} \sum_{n} n \Delta \theta \frac{\cos(n \theta)}{\sin(\theta)} - 1 \right)
$$

(6)

Where $\alpha=0$, because the planform and the twist stay constant.

Thus,

$$
delta \Delta C_{p} = 4 \cos(\alpha) \left( \frac{\alpha}{\sin(\alpha)} \sum_{n} n \Delta \theta \frac{\cos(n \theta)}{\sin(\theta)} - 1 \right)
$$

(7)

Here $\Delta \theta_{n}$ describes the difference in camber between the two sails. By adding this difference in camber to the “Sail Start” a new shape can be obtained. Equation (7) thus provides a way of modifying the “Sail Start” as a function of the difference between the two pressure maps. The new shape (“Sail Temporary”) has a pressure distribution that is closer to the desired solution (Figure 11, 12 & 13).
As can be seen from Figure 14, 15 & 16, the shape of this temporary sail resembles the shape of the original sail more closely than the shape of “Sail Start”.

Figure 14: Comparison at the ¼ span, between the sectional sail shapes

Figure 15: Comparison at the ½ span, between the sectional sail shapes

Figure 16: Comparison at the ¾ span, between the sectional sail shapes

A new sail whose shape is closer to the desired one has been created but there still remains a difference between the pressure maps. The process can now be repeated until the desired pressure distribution of the original sail is matched. This is an iterative process where at each loop the shape of the sail is modified until a shape which generates the same pressure map as the “target” sail is obtained.

4.3 Summary of the process

Figure 17 illustrates this iterative process schematically.

Figure 17: Summary of the process

Note that, at the beginning of the process, the “Sail Start” derives from the Inverse Thin Aerofoil Theory.
5. RESULTS AND DISCUSSION

The following results were found after 5 iterations. Figure 18, 19 & 20 are sectional views of the three pressure maps (from Sail Start, Sail Temporary and the Original Sail). At the beginning of the process the pressure map of Sail Temporary starts on the dotted line (pressure map of Sail Start) and, after each iteration, moves closer to the dashed line (Original Sail).

![Figure 18: Comparison at the ¼ span, between the ΔCp after 5 iterations](image1)

![Figure 19: Comparison at the ½ span, between the ΔCp after 5 iterations](image2)

![Figure 20: Comparison at the ¾ span, between the ΔCp after 5 iterations](image3)

Figure 21, 22 & 23 show cuts at different positions along the span. Like previously, the sections of Sail Temporary start on the dotted lines. At each iteration, the Δcamber is added and the generated new Sail Temporary moves closer to the Original Sail.

![Figure 21: Comparison at the ¼ span, between the sectional sail shapes after 5 iterations](image4)

![Figure 22: Comparison at the ½ span, between the sectional sail shapes after 5 iterations](image5)

![Figure 23: Comparison at the ¾ span, between the sectional sail shapes after 5 iterations](image6)
The following graphs show the differences between the Z values of the Original Sail and of the “Sail Start”. The results are presented as absolute values.

A sail designer considers a flying shape to be identical when the variation in camber is around 0.1 - 0.2% of chord length. In Figure 25, we note that the biggest difference is 0.006 m. This corresponds to a difference of 0.25% chord. We can thus confirm that the code produces accurate results within the normal tolerances used in sail design.

Taking into account the iterative approach, the authors expected that a perfect fit would result after many iterations. Hence the code was run with many more iterations (up to 100). The results showed that the shape didn’t change much after the first few iterations. This can be explained by the fact that the sail is modified only along the chord, so there is no relationship between the changes in sectional camber and the changes in span-wise curvature. Indeed, there are several ways to change the pressure distribution along one section:

- Change the camber of the section itself
- Change the camber of the neighbouring sections.
- Do both of the above.

Thus we can understand that the code may not produce a perfect match.

The code described in this paper used a VLM to generate the three-dimensional pressure map of the sail, it should however also be possible to couple the code with a RANSE-solver and carry out a similar process.

6. CONCLUSIONS

The goal of this paper was to find a three-dimensional inverse sail design method. That requirement has been met using inverse thin aerofoil theory and an iterative approach which, thanks to an extension of the inverse thin aerofoil theory, modifies a sail until it generates the desired pressure map.

Because an iterative process has been used, this method is not an analytical inverse method. A sail shape can be generated by using only its pressure map, plan-form, and twist.

The advantage of working in 3D is that the vertical flow component due to the tip vortices and the heel is taken into account.

Further work is required to link the code presented here and the span-wise optimisation routine developed by Junge [1].

Acknowledgements

The first author would like to express his sincere thanks to the Yacht Research Unit.
REFERENCES


