PCSAIL, A Velocity Prediction Program for a Home Computer

David E. Martin, Martin Consulting
Robert F. Beck, University of Michigan

ABSTRACT

An Excel Velocity Prediction Program has been developed to allow for rapid evaluation of yacht performance at the initial design stage. The required input consists of only the basic hull and sail dimensions. Empirical equations, based on these basic dimensions, are used for initial estimates of required hull parameters. As the design progresses the user can easily replace these default values with refined estimates or actual values. Because of its simplicity, and short turn around time, the program has been used as a teaching aid at the University of Michigan. Reconstruction of the program, PCSAIL, may be made with equations and other information provided in the Appendix.

The Excel "Solver" has been found to be a reliable means of finding the equilibrium boat speed and heel angle. It seeks the maximum boat speed by adjusting the sail flattening factor, F, and reef, R, and the lateral location of the "movable crew." In the case of a hinged centerboard, or dagger- board, it will also adjust the draft for maximum boat speed.

For sloop rigs the program will also take in the jib and set the spinnaker, at the appropriate wind angle, in order to gain maximum boat speed. The program plots the speed "polar," and velocity made-good, and determines the tacking angles.

NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>sail aspect ratio</td>
</tr>
<tr>
<td>( \beta_a )</td>
<td>apparent wind angle</td>
</tr>
<tr>
<td>CE</td>
<td>height of sail force center of effort</td>
</tr>
<tr>
<td>F</td>
<td>sail flattening factor</td>
</tr>
<tr>
<td>( F_{\text{avg}} )</td>
<td>average freeboard</td>
</tr>
<tr>
<td>R</td>
<td>sail reef factor</td>
</tr>
<tr>
<td>CRARM</td>
<td>position of movable crew</td>
</tr>
<tr>
<td>( \text{CRARM}_{\text{max}} )</td>
<td>maximum crew arm</td>
</tr>
<tr>
<td>T</td>
<td>draft</td>
</tr>
<tr>
<td>( T_c )</td>
<td>draft of canoe body</td>
</tr>
<tr>
<td>L</td>
<td>total aerodynamic lift</td>
</tr>
<tr>
<td>D</td>
<td>total aerodynamic drag</td>
</tr>
<tr>
<td>( S_c )</td>
<td>wetted area, canoe body</td>
</tr>
<tr>
<td>( M_r )</td>
<td>heeled righting moment</td>
</tr>
<tr>
<td>( M_s )</td>
<td>sail force heeling moment</td>
</tr>
<tr>
<td>( R_{\text{m2}} )</td>
<td>righting moment per degree at 2 degrees</td>
</tr>
<tr>
<td>( R_{\text{tot}} )</td>
<td>total hydrodynamic drag</td>
</tr>
<tr>
<td>( R_f )</td>
<td>frictional resistance on hull and appendages</td>
</tr>
<tr>
<td>( R_r )</td>
<td>residuary resistance on canoe body</td>
</tr>
<tr>
<td>( R_i )</td>
<td>induced resistance</td>
</tr>
<tr>
<td>( R_h )</td>
<td>added resistance due to heel</td>
</tr>
<tr>
<td>( R_{\text{waves}} )</td>
<td>added resistance due to sea waves</td>
</tr>
<tr>
<td>( \text{GM} )</td>
<td>meta-centric height</td>
</tr>
<tr>
<td>( \text{GZ} )</td>
<td>righting moment arm</td>
</tr>
<tr>
<td>( \text{GZ}_v )</td>
<td>righting moment arm at boat speed, ( V )</td>
</tr>
<tr>
<td>( U_a )</td>
<td>apparent wind at height of CE</td>
</tr>
<tr>
<td>( A_m )</td>
<td>area of main sail</td>
</tr>
<tr>
<td>( A_j )</td>
<td>area of jib</td>
</tr>
<tr>
<td>( A_s )</td>
<td>area of spinaker</td>
</tr>
<tr>
<td>( A_y )</td>
<td>area of mizzen</td>
</tr>
<tr>
<td>( A_{ys} )</td>
<td>area of mizzen staysail</td>
</tr>
<tr>
<td>( C_l )</td>
<td>sail lift coefficient</td>
</tr>
<tr>
<td>( C_d )</td>
<td>total aerodynamic drag coefficient</td>
</tr>
<tr>
<td>( C_{dp} )</td>
<td>sail parasitic drag coefficient</td>
</tr>
<tr>
<td>( C_i )</td>
<td>induced areao drag coefficient</td>
</tr>
<tr>
<td>( C_{dc} )</td>
<td>hull &amp; mast &quot;windage&quot; drag coefficient</td>
</tr>
<tr>
<td>( Q )</td>
<td>aero-stagnation pressure, ( 1/2 \rho_a V_t^2 )</td>
</tr>
<tr>
<td>( V_t )</td>
<td>true wind velocity</td>
</tr>
<tr>
<td>( V_c(z) )</td>
<td>true wind at height z</td>
</tr>
<tr>
<td>( z )</td>
<td>height above water</td>
</tr>
<tr>
<td>( V )</td>
<td>velocity of boat along course</td>
</tr>
<tr>
<td>( F_{\text{drive}} )</td>
<td>sail driving force</td>
</tr>
<tr>
<td>WPA</td>
<td>water-plane area</td>
</tr>
<tr>
<td>h</td>
<td>depth of hydro-side force below water</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>yacht weight displacement</td>
</tr>
<tr>
<td>( V_c )</td>
<td>volume displacement of canoe body</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>air density</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>water density</td>
</tr>
<tr>
<td>( \beta_t )</td>
<td>true wind angle</td>
</tr>
<tr>
<td>( \phi )</td>
<td>heel angle</td>
</tr>
<tr>
<td>( \theta )</td>
<td>trim angle</td>
</tr>
</tbody>
</table>

Subscripts m, j, s, and y refer to the main, jib, spinnaker and mizzen sails respectively.
INTRODUCTION

Velocity prediction programs are not usually written in languages that are widely used on home computers. Most home computers only have Excel or Lotus 1-2-3. A typical VPP may also require input of offset tables or include lines development programs. Both require a considerable level of effort before the first evaluation of an initial design may be made. Thus, it is believed that a simplified Excel, Lotus 1-2-3 or Quattro Pro might find wide use on home computers. All of these include a "Solver", which can be used to find an optimum solution to the standard force and moment equilibrium equations of a VPP. All three employ optimizers that are based on the nonlinear "solver" supplied by Frontline Systems Inc.

The purpose of this paper is to describe an Excel velocity prediction program that, together with a number of empirical equations, allows for the rapid evaluation of an initial design, based on only the basic hull and sail dimensions. Because it is simple, and may be used on a home PC, it has been used at the University of Michigan as a teaching aid.

Initial estimates of the required hull parameters, such as displacement, are gained by means of empirical equations that describe these hull parameters in terms of the basic hull dimensions. The estimates are based on a range of data taken from the public domain, including US Sailing's profile of the North American IMS fleet (US Sailing, 1999). They are presented here along with their correlation coefficients where possible. As the design progresses these initial estimates can easily be replaced by refined estimates, or actual values.

The method of sail force calculation suggested in Principles of Yacht Design (Larsson and Eliasson, 1994) is employed because it is simple and quite flexible. The method provides for ready determination of lift and drag coefficients as continuous functions of the apparent wind angle based on only five paired values at five apparent wind angles. The program also provides for the ready application of IMS sail force coefficients (Claughton, 1999), the use of the Larsson/Eliasson coefficients, or the substitution of a user's values.

The VPP establishes equilibrium between the sail force equations and the hydrodynamic resistance equations based on the Delft Systematic Yacht Hull Series, (DSYHS). These hydrodynamic equations are fully detailed and also readily available in the publications of the Chesapeake Sailing Yacht Symposiums (Gerritsma et al., 1993), and more recently, (Keuning and Sonnenberg, 1999).

PCSAIL was developed on a home PC. It is simple to use and also quite flexible, allowing for use input of both data and resistance equations. Because of this utility and flexibility, and because it can be used by anyone having Excel, it is hoped that it may be useful to yacht designers, teachers, and those having a general interest in sailing technology.

THE VELOCITY PREDICTION PROGRAM

Hydrodynamic Equations

The Delft Systematic Yacht Hull Series (DSYHS), collected towing tank data on a wide range of model hull proportions, speeds, heel and leeway angles. The data were then curve fitted to provide estimates of resistance, yaw angle, wetted surface, and dynamic righting moment. The detailed equations from this work have been published in the 11th and 14th Proceedings of the CSYS, (Gerritsma et al., 1993, Keuning, 1999). The equations of Gerritsma et al. are detailed in the Appendix, and will be reviewed here in outline form.

The total hydrodynamic resistance, \( R_{tot} \), may be taken as:

\[
R_{tot} = R_f + R_r + R_r + R_h + R_{waves} + R_{prop} \tag{1}
\]

where:

- \( R_f \) = frictional resistance on the hull and appendages
- \( R_r \) = residuary resistance on the canoe body
- \( R_h \) = induced resistance
- \( R_h \) = added resistance due to heel
- \( R_{waves} \) = added resistance due to sea waves
- \( R_{prop} \) = resistance of propeller

In PCSAIL, the first four elements are defined by the equations suggested by Gerritsma, 1993. The added resistance due to sea waves is based on the equations suggested in the 14th CSYS (Claughton, 1999) which are in a readily usable form. However, the program also provides for the substitution of the user's own equation for this resistance increment. Propeller resistance is from Larsson and Eliasson. All of the detailed equations may be found in the Appendix.

Gerritsma et al. also describe the heeled righting moment, \( M_r \), in terms of the metacentric height, \( GM \), and a dynamic increment, \( MN \), (usually negative) due to the yacht's speed, \( V \). The righting moment arm, \( GZ_r \), at boat speed \( V \), and righting moment, \( M_r \) are given by:

\[
GZ_r = (GM + MN) \sin \phi \tag{2}
\]

where:

- \( MN \) = a function of the boat speed defined in the Appendix

\[
M_r = GZ_r \Delta + M_{rec} \tag{3}
\]

\( M_{rec} \) is the righting moment from the movable crew defined in the appendix.
It should be noted that the resistance equations, (Gerritsma, et al., 1993) have been modified to include a center or dagger-board. These modifications adopt the concept of equivalent draft developed by Kerwin, 1978, for keel-centerboard boats. The board draft has been included as one of the variables to be optimized for determination of the maximum boat speed. The resistance equations also add the frictional resistance and form drag of a keel bulb according to the formulation of Keuning and Binkhorst, 1997. The details for both additions are included in the Appendix.

Sail Lift and Drag Forces

At the equilibrium speed and heel angle the hydrodynamic resistance of the hull must balance the sail driving force and the righting moment must balance the sail heeling moments. PCSAIL sail forces are based on the form of the equations suggested by Larsson and Eliasson, 1994. Sail forces are resolved into their lift, L, and drag, D, components using the conventional relations:

\[ L = C_L \frac{1}{2} \rho U_a^2 A_n \]  
\[ D = C_D \frac{1}{2} \rho U_a^2 A_n \]  

where:

- \( C_L \) = lift coefficient for the sail combination in use, based on \( A_n \)
- \( C_D \) = drag coefficient for sail combination in use, based on \( A_n \)
- \( A_n = A_f + A_m + A_y \)  

\( A_f = \) nominal area of fore-triangle = 0.5\( PI \)
\( A_m = \) nominal area of main sail = 0.5\( PE \)
\( A_y = \) nominal area of mizzen = 0.5\( PY \)
\( U_a = \) the effective apparent wind velocity at the height of the center of effort, \( CE \), of the sail combination in use (see below)

I, J, P, E, P_y and E_y are defined in FIG. 2.

The apparent wind velocity, \( U_a \) and angle \( \beta_a \) are defined in a plane through the heeled mast. It is a function of the height of the center of effort, \( CE \), the heel angle, \( \phi \), the true wind velocity at a height of 10 meters above the water, the wind gradient with height and the true wind angle. This function is complex, includes the boat velocity, and is detailed in the Appendix.

The resultant \( CE \) is found by the following:

\[ CE = \frac{A_m CE_m + A_f CE_f + A_y CE_y + A_m CE_m + A_f CE_f}{(A_m + A_f + A_y + A_m)} \]  

where:

- \( A_m = \) nominal area of main = \( P E/2 \)
- \( A_f = \) actual area of jib = 0.5 \( LP \) \( (I^2 + J^2)^{1/2} \)
- \( A_y = \) nominal area of mizzen = 0.5 \( P Y \)
- \( A_y = \) area of distressed staysail = 0.5 \( YSD \) (\( YSMG + YSF \))

\( CE_m = 0.39 P + BAD \)
\( CE_f = 0.39 I \)
\( CE_y = 0.39 P_y + BADY \)
\( CE_y = 0.39 P_y + BADY \)

where: \( SL = \) spinnaker leech, \( P = \) main hoist, \( BAD = \) boom above deck. \( YSD \), \( YSMG \), and \( YSF \) are the height, midgirth, and foot of the mizzen staysail, (FIG. 2). It should be noted that in equation (7) the spinnaker area is based on Larsson and Eliasson, 1994, and is not the same as that used by IMS. IMS uses:

\[ A_s = 0.6 \) (min rated luff) (min. rated max. width)

PCSAIL is programmed to use the Larsson/Eliasson spinnaker sail area, rather than the IMS value. The PCSAIL output data, for the boats evaluated in this study, have not been found to be significantly different if either is used. However, since in this study a comparison of PCSAIL output is made with IMS VPP output, the IMS spinnaker area was used in all cases presented.

The total drag coefficient, \( C_D \) in equation (5) is determined as the sum of three separate components:

\[ C_D = C_{dp} + C_{di} + C_{do} \]  

where:

- \( C_{dp} = \) parasitic or viscous drag coefficient
- \( C_{di} = \) induced drag coefficient
- \( C_{do} = \) windage drag coefficient

As given by airfoil theory, the induced drag is proportional to the lift coefficient squared, and inversely proportional to the aspect ratio. It may be approximated by:

\[ C_{di} = C_i^2 \frac{1}{AR} + 0.005 \]  

where:

\( AR = \) Aspect Ratio = \( \lambda (EHM + F_{ave}) \)/\( A_n \)
\( \lambda = \) rig height factor
\( \lambda = 1.1 \) \( \beta_a < 30 \) deg
\( \lambda \) linearly declines to 1.00 at \( \beta_a = 90 \) deg
\( F_{ave} = \) average free board
\( EHM = \) mast height above deck

The windage drag coefficient accounts for the windage on the hull and rig and is given by:
\[ C_{do} = 1.13\left( BF_{avg} Q_{rel}\right) +(EHM \cdot EMDC)/A_n \]  

where:

- \( EMDC = \text{avg. mast diameter} \)
- \( Q_{rel} = U_r^2/U_a^2 \)
- \( U_r = \text{wind velocity relative to hull.} \)
- \( U_a \) at \( z = F_{avg}/2 \)
- \( U_a \) at \( z = \text{CE} \)

This formulation of \( C_{do} \) differs from the Larsson/Eliasson treatment, which does not include the \( Q_{rel} \) term. This term is suggested by Claughton's, 1999, treatment of \( \text{value at the height of the sail center of effort.} \)

Dynamic pressure at the height of the hull is much less than adapted to the Larsson method because at wind speeds of 20 knots, and at low apparent wind angles, the hull drag appeared to reduce the boat speed significantly. Because this study does not provide a direct means of evaluating this adaptation, the program provides for the ready substitution of a user's equation for the windage drag coefficient, in an open cell, that will be called only upon the user's instruction.

The lift coefficient and parasitic drag coefficients are determined by summing the individual components of each sail. Since the \( C_l \) and \( C_{dp} \) coefficients are based on the nominal area, \( A_n \), the individual components must be appropriately normalized as:

\[ C_l = \left( C_{lam} A_m + C_{la} A_l + C_{ly} A_y + C_{lyr} A_{yr} \right) / A_n \]  
\[ C_{dp} = \left( C_{dpm} A_m + C_{dp} A + C_{dpa} A + C_{dpa} A + C_{dpa} A \right) / A_n \]  

The user has several choices to determine the individual components for each sail. For a two-masted vessel, the user can select the five sets of sail force coefficients suggested by Larsson and Eliasson, 1994, (Tables A-II and A-III, of the Appendix). The Larsson/Eliasson method can also be applied to a sloop by setting the dimensions of the mizzen sails and mast to zero. For a sloop, the user can also use the sail force coefficients suggested by Claughton, 1999. Estimated values from Claughton's curves are shown at the same five apparent wind angles in Tables A-IV and A-V (Appendix). Claughton's equations for determining the overall lift and drag coefficients are the same as (11) and (12) except that he multiplies each individual sail term by a blanketing factor, \( B_i \). Since he says that these blanketing terms are usually taken as unity, equations (11) and (12) are used directly in PCSAIL with Claughton's coefficients.

A third possibility for advanced users is to enter his or her own values for \( C_l \) and \( C_{dp} \). These values can be entered directly in the sail force coefficient array on the SAILPARAM spreadsheet. The Larsson/Eliasson values can be reentered by clicking on a button on this same sheet.

In either case the "blended", or total sail force coefficients will be computed from the selected values for individual sails (for five apparent wind angles) using equations (11) and (12). To define the sail force coefficients as continuous functions of the apparent wind angle, cubic spline functions are fitted through values at these five apparent wind angles. The equations for this may be found in Numerical Recipes (Press et al., 1992). In order to determine the coefficients of a cubic spline fit through five paired values it is necessary to solve four equations simultaneously. In PCSAIL, Gauss elimination is used.

These cubic spline equations provide the necessary continuous functions for the lift and parasitic drag coefficients. As previously noted, in the case of drag, the parasitic component must be added to the induced drag and the drag of the mast and hull. These are not included in the cubic spline fits because they are computed directly.

The effects of sail flattening, \( F \), and reefing, \( R \), are handled as follows:

- **Flattening**: Multiply \( C_l \) by flattening factor \( F \)
- **Reefing**: Multiply \( C_l \) and \( C_{dp} \) by reef factor squared, \( R^2 \), multiply height of CE by reef factor, \( R \).

### Sail Driving and Heeling Forces

The sail forces are are given in terms of lift, \( L \), and drag, \( D \), components by (4) and (5), which by definition are components normal to, and parallel to the apparent wind direction. VPP predicts the velocity of a yacht by finding the velocity at which the hydrodynamic resistance is in balance with the sail driving force, and the heeled righting moment balances the sail force heeling moment. Since the hydrodynamic drag is parallel to the yacht's velocity, the balance is done in an axis system moving along the yacht's course. The aerodynamic forces are resolved into a driving force parallel to the course and a heeling force normal to the mast by:

\[ F_{drive} = L \sin \beta_a - D \cos \beta_a \]  
\[ F_{heal} = L \cos \beta_a + D \sin \beta_a \]  

where:

- \( \beta_a \) is the effective apparent wind angle defined in the Appendix.

These forces can be used in the basic equilibrium equations for a rigid body.

### Equilibrium Equations

The basic equilibrium equations for a rigid body are as follows:

\[ \Sigma F_x = 0, \ x \text{ direction is boat velocity, } V \]  
\[ \Sigma F_y = 0, \ y \text{ direction is horizontal and normal to } V \]  
\[ \Sigma F_z = 0, \ z \text{ direction is vertical} \]  
\[ \Sigma M_x = 0 \]  
\[ \Sigma M_y = 0 \]  
\[ \Sigma M_z = 0 \]
The sum of the vertical forces, \( \Sigma F_z \), is assumed to be satisfied by a balance of the yacht's weight with the buoyancy forces. And the moment balance about the z-axis is assumed to be satisfied by mast and keel placement, and sail trim.

The force balance in the y-direction is accomplished by having a leeway angle that will generate hydrodynamic side forces that will balance the aerodynamic side force. The moment balance about this axis is assumed to be automatically satisfied by a small trim angle about the y-axis that generates a longitudinal righting moment to balance the sail force moment which tends to set the bow down. This bow down trim is usually small and not taken into consideration. However the resistance equations of Keuning and Sonnenberg, 1999, do include estimates of the increased hydrodynamic resistance due to the trim angle.

Equilibrium of the forces in the x-direction requires that the sail driving force, \( F_{drive} \), is in balance with the total hydrodynamic drag, \( \text{R}_{tot} \). Equilibrium of the moments about the x-axis requires that the heeling moment equals the righting moment. In other words:

\[
F_{drive} = \text{R}_{tot} \tag{15}
\]

\[
M_{heel} = M_{right} \tag{16}
\]

The equations defining these forces and moments are cumbersome, as may be seen in the Appendix. They are direct functions of \( V \) and \( \phi \) and the hull and sail geometry, true wind speed and true wind direction. They indirectly also depend on the variables \( R \), \( F \), \( \text{CRARM} \), and \( T \) because the sail forces depend on \( R \) and \( F \) and the hydrodynamic characteristics depend on \( \text{CRARM} \) and \( T \). For fixed values of \( R \), \( F \), \( \text{CRARM} \), and \( T \) the VPP finds the equilibrium \( V \) and \( \phi \) that satisfy (15) and (16). However, this solution does not necessarily yield the maximum \( V \) that is possible. To find the maximum \( V \), it is necessary to also allow \( R \), \( F \), \( \text{CRARM} \), and \( T \) to vary. The problem becomes one of finding the optimum combination of \( R \), \( F \), \( \text{CRARM} \), and \( T \) and \( \phi \) that maximize \( V \) subject to appropriate constraints on these variables.

**Optimization**

In PCSAIL, the optimization of \( R \), \( F \), \( \text{CRARM} \), \( T \) and \( \phi \) for the maximum boat velocity is accomplished by means of the Excel "Solver" which is supplied by Frontline Systems (http://www.frontsys.com). It employs a Generalized Reduced Gradient method described by Lasdon et al., 1978. The "Solver" satisfies the equilibrium equations by means of a constraint requiring that the difference between the sail driving force and the total resistance, \( \text{DLT}_{drive} \), be less than 0.05 percent, and that the difference between the sail heeling moment and the heeled righting moment, \( \text{DLT}_{heel} \), also be less than, or equal to 0.05 percent:

\[
(F_{drive} - \text{R}_{tot})/\text{R}_{tot} \leq 0.0005
\]

\[
(M_{heel} - \text{M}_{right})/\text{M}_{right} \leq 0.0005
\]

In seeking values for \( R \), \( F \), \( \text{CRARM} \), and \( T \), which maximize the boat speed, these variables are also constrained.

The use of the "Solver" is relatively straightforward and can readily be understood by consideration of the following table, which is similar to the "Solver" dialog box.

<table>
<thead>
<tr>
<th>Solver Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set Target Cell</td>
</tr>
<tr>
<td>( V/2 )</td>
</tr>
<tr>
<td>Equal to</td>
</tr>
<tr>
<td>( O \ Max )</td>
</tr>
<tr>
<td>( O \ Min )</td>
</tr>
<tr>
<td>( O \ Value )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>By Changing Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( F )</td>
</tr>
<tr>
<td>( R )</td>
</tr>
<tr>
<td>( \text{CRARM} )</td>
</tr>
<tr>
<td>( T )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subject to Constraints (example constraints shown):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V \leq 20 )</td>
</tr>
<tr>
<td>( V \geq 3 )</td>
</tr>
<tr>
<td>( \phi \leq 35 )</td>
</tr>
<tr>
<td>( \phi \geq 0.01 )</td>
</tr>
<tr>
<td>( F \leq 1.0 )</td>
</tr>
<tr>
<td>( F \geq 0.2 )</td>
</tr>
<tr>
<td>( R \leq 1.0 )</td>
</tr>
<tr>
<td>( R \geq 0.4 )</td>
</tr>
</tbody>
</table>

**Table I. Solver Input**

Initial guesses for these variables are entered and the solver then seeks by iteration the maximum boat speed, and returns the values, for the variables, which yield that maximum. The "target" is taken as \( V/2 \) because the solver will not accept one of the variables for change as the target.

**Required and Optional Input**

Table II lists both the required and optional input for PCSAIL, and indicates the figures into which the basic dimensional input is made.

Figure 1 is taken from the Excel worksheet into which the required hull dimensions for a boat with a fixed keel are entered. Required data are entered in the heavily underscored cells, (colored green for ready identification on the computer screen). By scrolling down the user will also find cells for entrance of center and dagger-board dimensions, including a minimum sweep angle.

The HULLPARAMETER Spreadsheet, not shown, provides cells for the entrance of required data:

1. Whether the hull has dead-rise, (YES/NO)
2. The number of crew, and "movable" crew.
Required Information

See Figures 1 & 2

BOAT ID, Number of Crew, Number movable
Crew, CB; YES/NO, DB; YES/NO
Separate Rud?, YES/NO, Deadrise?, YES/NO

Required Basic Hull Dimensions

See Figure 1

Lw, Lb, B, F, T, Tc, CB, CBm, DBm
Rudm, Kpm, Dbm
(Lin = min CB sweep angle)

Required Basic Sail Dimension

See Figure 2

P, E, Ep, I, J, LPG, BAD, Bady, YSF, YSMG, YSD

Optional Sail Dimensions

EHM, SI, EMDC

Optional Hull Parameters

Δ, RM, Cp, Ld, Lh, Sh, WPA,
A/R, A/P, DBTHK, CBTHK,
THK1, THK2

Table II. VPP Input

If hull parameters such as the prismatic coefficient, Cp, are available they can be entered in cells that are underscored with dashed lines. If they are not yet available these cells must be left empty, (or zero entered), and the default values will be entered using empirical equations. The empirical equations, source data, and R² values are listed in Table A-I (Appendix). For example, the total hull displacement empirical equation is given by:

\[ \Delta = (0.0008(LwB Tc)^2 + 13.317 LwB Tc + 926.78) \]

\[ R^2 = 0.94 \]

N.A. IMS Fleet (18)

Note: Foot note re variables under Table A-I

Figure 2 is taken from the SAILINPUT spreadsheet and shows outlined cells into which the basic sail and mast dimensions are entered. The cells with dashed underscoring are for optional input.

Operation of the VPP

The following describes the operation of the program, including the "macros" that automatically run the solver for a series of wind directions. The program automatically plots the boat speed, velocity made-good, and determines the best tacking angles.

Operation of the program is by use of the "RUNIT" spreadsheet shown in Figure 3.

Since the program can be operated with either the resistance equations of Gerritsma et al., 1993, or Keuning and Sonnenberg, 1999, it is first necessary to specify which total resistance is desired, Rtot93, or Rtot99 respectively. The program is not allowed to run with Rtot99 if the boat has a centerboard or dagger-board because the equations of Keuning et al. (for keel residuary resistance) were not developed using experimental data which included such configurations.

Fig. 1 HULL INPUT Spreadsheet
Run Time Variables
The basic run time variables are the true wind direction and true wind speed, which are entered in the "RUNIT" spreadsheet. The user may also determine what sails to use in the manual operation mode. This choice may be made by directly entering the choice in the cell identified as "SAILSET." Three choices are available.
1. By entering "MJ" the program will run with just the main and jib, using Claughton's sail force coefficients.

2. By entering "MJS" the program can run with first the main and jib and then take in the jib and set the spinnaker when the VPP determines that this will yield a higher boat speed. The program will use the Claughton sail force coefficients when in this mode.

3. By entering "ALL" the program can run with all five sails set as indicated in the Larsson, 1994, approach, and the program will normally use the Larsson/Eliasson coefficients. However, as will be noted, it will also be possible to run with the user's coefficients. By this selection the user can also evaluated a sloop rig simply by entering zero values for the mizzen dimensions.

The solution for a single wind condition can be found by entering the initial "guesses" for boat speed, heel, crew-arm, and draft, (V,cj,CRARM and T). If there is a center or dagger-board, the initial draft "T" must lie between the upper and lower constraints for T. In Excel the "Solver" is called (under "Tools"), and simply "run." If the initial guesses are reasonable, a maximum boat speed is found by iteration and the best values therefore returned to the cells into which the initial "guesses" were entered. The equilibrium values for the Delta Drive and Delta Mom will also be returned. At equilibrium speed and heel these values should be not greater than the constraints set into the solver to ensure that the sail driving force is in balance with the total resistance, here 0.0005, or 0.05 percent. They are recorded for each wind direction on the "RUN" spreadsheet so as to allow the user to readily verify that each solution is valid.

Automatic Running Features

Macros have been written to simplify operation of the VPP. Initial "guesses" of the boat speed, heel, reef, flat etc are automatically introduced for a true wind direction of 38 degrees. These are based on the yacht's basic "hull speed" and wind strength. The user can, by clicking one of seven "buttons," run the program in seven modes:

1. At a single entered wind velocity, and a single true wind direction. In this mode the program will select SAILSET main and jib, MJ, for a sloop, and SAILSET "ALL" if the vessel is two masted.

2a Automatic running of all 23 preset wind directions, at a single user selected wind speed for sloops. With the just the main and jib, MJ, using Claughton's sail force coefficients.

2b Automatic running of all 23 selected wind speeds and all 23 wind angles for sloops. With just the main and jib, (MJ) and using the Claughton sail force coefficients.

3a Automatic running of all 23 wind directions at one selected true wind speed for sloops, with SAILSET "MJS". With SAILSET "MJS" the program runs first with just the main and jib, and then automatically takes in the jib and sets the spinnaker as needed to gain the maximum boat speed. The Claughton sail force coefficients are used for main, jib and spinnaker.

3b Automatic running of all five true wind speeds and at all 23 preset wind angles, again with SAILSET "MJS" which sets the spinnaker as needed for maximum speed. Claughton's coefficients are used.

4a. Automatic running, at a single wind speed, of all preset wind directions for two masted vessels. SAILSET "ALL", and Larsson sail force coefficients will be used in this automatic mode. The user has an option of entering his or her sail force coefficients.

4b. Automatic running of all 5 wind speeds and all 23 preset wind directions for two masted vessels. SAILSET "ALL" and Larsson sail force coefficients will be used in the automatic mode. As above the user can substitute his or her coefficients.

In some instances it was found that the macro, (as initially written) would not yield valid solutions, particularly with center or dagger-boards. With a board, the maximum velocity could be found for the board in the wrong condition, not yielding the true maximum. This can occur, for example, when the frictional resistance on the boards is close to the induced resistance. Since the macro starts from a true wind direction of 38 degrees, draft iteration will first be from the board full down. Accordingly, the macro was modified to include a step that examined the maximum boat speed for the draft being set initially at its lower constraint, Tlow. This is called for if the frictional resistance on the board approaches the magnitude of the induced resistance. If the value of the board friction resistance divided by the induced resistance, Rfrds/Ri is greater than 0.95 the macro calls for Tlow and restarts the "Solver." It then selects the maximum from the solution starting with the higher T, and that starting with Tlow.

A similar provision is included in the macro that calls for a "nudged" initial velocity if a valid solution is not found which starts from the velocity solution of the preceding wind angle. This is the reason for a cell labeled "NUDGE PERCNT". The user can enter a value or allow it to take the default value of 5 percent. The macro records the last velocity solution in a cell, and will call values 5 percent above and then 5 percent below this value if a valid solution is at not, at first, found. This problem has been infrequent.

Custom Running of PCSAIL

It is anticipated that potential users of PCSAIL would be concerned that a design "improvement" might be
too dependant upon the selection of sail force coefficients. Thus the user is provided the option of entering his or her own coefficients into the Larsson/Eliasson array. And a cell has been reserved for user entrance of an alternate equation for the “windage” drag of the hull and rigging. Such changes in coefficient values, or the “windage’ equation can be made in the indicated cells in the SAILPARAMETER spreadsheet. Unless these changes are entered, the program will be run as previously indicated.

PCSAIL also provides for automatic recording of the above solutions. The following shows samples of typical computations, and the automatic recording thereof.

RESULTS

All of the following examples were from computations employing the equations of Gerritsma et al. 1993, modified as previously noted to consider centerboards, keel bulbs and sea waves. None of the following results were based on the equations which included the residuary resistance of the keel, (Keuning and Sonnenberg, 1999). Results are shown for the Ranger 26, C&C35MKI, CAL40, Swan46BK (bend keel), and the Saber 42CB, a centerboard boat. All, results were obtained using the hull parameters from the IMS VPP outputs, except for those shown in Figure 19 which includes results using the default estimates of hull parameters.

Figure 4, below, is a typical PCSAIL output sheet. These example data are for a Ranger 26 in a true wind of 20 knots. It shows for each of the indicated true wind angles the various recorded out-puts; boat speed, heel, flat, reef, effective apparent wind velocity, effective apparent wind angle, velocity made good, draft, and the SAILSET, here main and jib, MJ.

It may be noted that the boat speed, in knots, has been rounded to four decimal places. This was done, in part, because the table look-up feature of Excel requires a unique value for seeking the best tacking angle. In a number of instances the “repeatability” of the “Solver” has been tested by reversing the sequence of solutions in order to determine if the result would depend on the initial velocity “guess” being above or below the solution. The macro starts at a true wind angle of 38 degrees and progresses through 178 degrees, in each instance taking the values from the preceding solution as initial values for the next. When the sequence was reversed, starting at a true wind angle of 178 degrees and progressing to 38 degrees, the boat speeds were found to repeat to five places.

In the following figures PCSAIL data are shown as trend lines and IMS data for the same boat are shown as the individual data points. For comparison with IMS data, PCSAIL used IMS spinnaker areas.

Figure 5 shows a plot of a succession of results for a Ranger 26 at three true wind speeds: 6, 12 and 20 knots. The agreement between the output of PCSAIL and the IMS prediction is good. Here the SAILSET was MJ, first main and jib, and then main and spinnaker.

![FIG. 5 Ranger 26 Using SAILSET MJ](image-url)

| Tru Wind | VPP Data | VPP Data | VPP Data | VPP Data | VPP Data | VPP Data | SAILSET | Tru Wind | VPP Data | VPP Data | VPP Data | VPP Data | VPP Data | VPP Data | SAILSET | Tru Wind | VPP Data | VPP Data | VPP Data | VPP Data | VPP Data | VPP Data | SAILSET |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 20       | 38       | 4.3828   | 12.081152 | 0.499022 | 0.832225 | 38.3008328 | 0.473811 | SAILSET  | MJ       | 38.3008328 | 0.473811 | SAILSET  | MJ       | 38.3008328 | 0.473811 | SAILSET  | MJ       |
| 40       | 4.8147   | 12.04512 | 0.563323 | 0.951271 | 38.4785492 | 0.432498 | SAILSET  | MJ       | 38.4785492 | 0.432498 | SAILSET  | MJ       | 38.4785492 | 0.432498 | SAILSET  | MJ       |
| 60       | 5.1880   | 12.01012 | 0.579345 | 0.958656 | 38.5450171 | 0.406541 | SAILSET  | MJ       | 38.5450171 | 0.406541 | SAILSET  | MJ       | 38.5450171 | 0.406541 | SAILSET  | MJ       |
| 80       | 5.5377   | 12.16457 | 0.815560 | 0.889656 | 38.6111491 | 0.381841 | SAILSET  | MJ       | 38.6111491 | 0.381841 | SAILSET  | MJ       | 38.6111491 | 0.381841 | SAILSET  | MJ       |
| 100      | 5.1873   | 22.733548 | 0.851022 | 0.927498 | 38.5450171 | 0.334622 | SAILSET  | MJ       | 38.5450171 | 0.334622 | SAILSET  | MJ       | 38.5450171 | 0.334622 | SAILSET  | MJ       |
| 120      | 5.4641   | 25.114497 | 0.701177 | 0.869246 | 38.1273279 | 0.877777 | SAILSET  | MJ       | 38.1273279 | 0.877777 | SAILSET  | MJ       | 38.1273279 | 0.877777 | SAILSET  | MJ       |
| 140      | 5.9730   | 25.402931 | 0.748157 | 0.474055 | 35.7002002 | 0.998888 | SAILSET  | MJ       | 35.7002002 | 0.998888 | SAILSET  | MJ       | 35.7002002 | 0.998888 | SAILSET  | MJ       |

![FIG. 4 VPP DATA Spreadsheet for Ranger 26](image-url)

107
Figure 6, above, shows the same Ranger 26, but with the main and jib only for all true wind angles. This diagram again indicates favorable agreement with IMS results at all three wind speeds.

Figure 7 shows a typical automatic plot of Velocity Made Good, here for the Ranger 26 with main, jib, and spinnaker at wind speeds of 6, 12 and 20 knots.

Figures 8 and 9 show comparable results for the C&C35 MKI, again indicating favorable results with the IMS output.

Figures 10 and 11 show favorable agreement with IMS predicted speeds for the CAL 40.
Figure 12 shows a comparison of the PCSAIL speed predictions for a CAL 40 with those of IMS, but using SAILSET "ALL" and the Larsson/Eliasson sail force coefficients. The agreement with the IMS data is not as good as in figure 11. The difference clearly relates to differences in the sail force coefficients (Tables AII-AV Appendix).

These data, and a comparison of figures 10 and 11 make it evident that setting a spinnaker, a step change in sails, makes it difficult to construct a single continuous function for the lift and drag coefficients that will adequately treat the full range of true wind conditions. It should be noted that Larsson calls attention to this limitation, saying that this treatment is “quite crude,” but useful. It provides a means of treating two masted boats.

Figures 13 and 14 again show favorable agreement with the IMS predictions for a Swan46BK, (bulb keel) for SAILSET MJS, and SAILSET MJ.

Figure 15 presents PCSAIL output for the Saber42CB, a centerboard boat. Note in the draft column, that in a true wind of 20 knots, and a true wind angle of 108 degrees, the board is no longer fully extended.

Figures 16 and 17 show comparisons with IMS for the Saber42CB. Here the agreement off the wind is good, but close on the wind PCSAIL predicts a slower speed than IMS. The scope of this study does not provide a basis for determining the reason for the observed difference on the wind. But clearly, given agreement off the wind, the formulation for the equivalent draft, $T_{eq}$, is suspect. PCSAIL uses Kerwin's formulation because it is quite simple, requiring only DHKA, the minimum draft. The IMS formulation is more complex, requiring the longitudinal location of the tip of the fully extended centerboard and the effective depth of the hull above that location. A more detailed study, including more centerboard boats, is needed.
Table 1

<table>
<thead>
<tr>
<th>Boat Speed, Knots</th>
<th>True Wind Angle, Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

**SABER42CB, Main & Jib/ Main & Spin**

**FIG.16. Saber 42CB SAILSET MJ**

Figure 18 shows a sample of a Summary-Data spreadsheet, here for the C&C35MKI. It shows a complete set of boat speed and velocity made good for five wind speeds, and all wind angles. It also shows the tacking angles and average time per mile, (for three wind speeds) around a "circular" ten mile course.

Figure 17 showed PCSAIL's prediction of the speed performance of the Saber42CB having heel resistance, displacement etc based on the IMS certificate. Figure 19 shows these data in comparison to the performance of a default "Saber42CB" having just its basic sail and hull dimensions, and the hull parameters being determined by the empirical equations suggested herein. The agreement is not good because for this particular hull the predicted displacement was quite low; only 19,022 lbs, while the actual displacement of 24,988 lbs. These data illustrate the need for caution in using such default values, particularly those for displacement. The estimates are suggested only as a help in establishing an initial design. Experienced yacht designers will undoubtedly make adjustments to these parameters based on their particular design objectives. For example, since the estimated parameters will include $C_p$, $L_{wh}$, $B_{wl}$, $T_{cm}$, and the displacement, the program calculates the midship section, $C_m$. If this value is not consistent with the designer's intent, it will be easy to make suitable adjustments to various of the above dimensions and or displacement.

![FIG.17. Saber 42CB SAILSET MJ](image)
FIG. 18 Summary Data for C&C35MKI

SABER42CB IMS HULL PARAMETERS VS "SABER42CB" WITH DEFAULT PARAMETERS

FIG.19 Saber 42CB Having Default Parameters

All of the preceding output data were based on the total resistance, $R_{tot93}$, (Gerritsma, et al., 1993). A comprehensive study of comparable results using "$R_{tot99}$" has not been conducted. However verification of the VPP resistance output with the Keuning/Sonnenberg equations has been made. Additionally, for every velocity calculation, each resistance element for both the Gerritsma, and Keuning equations is calculated. These are recorded and may be observed if the next calculation is not initiated automatically. The total resistances are usually in good agreement. Differences between the two resistances have been observed in the case of the CAL 40, which has a large keel volume. This is not surprising since Keuning and Sonnenberg caution that the keel residuary resistance (Keuning and Sonnenberg, 1999) will not be reliable for all keel shapes since, "... the data set is still rather limited."

CONCLUSIONS

The solver used by Excel (and Lotus 1-2-3, and Quattro Pro) has been found to be a reliable tool for seeking optimum solutions to the equilibrium equations of a VPP. It finds the optimum condition of the crew location, reef, sail flattening, and centerboard adjustment (draft) that maximizes the boat speed while establishing the basic equilibrium conditions of a VPP.

The velocity predictions of PCSAIL, for the five boats presented herein, have been found to be in favorable agreement with IMS predictions for these same boats if the IMS sail force coefficients (Claughton, 1999) are employed. The agreement is not, and should not be expected to be exact because there are differences between this VPP and the IMS VPP, in both the aero and hydrodynamic equations. However the quantitative agreement is regarded as "good" and the shape of the speed versus true wind angle curves is thought to be in very good agreement with the IMS output.

Agreement with this limited IMS data does not imply that PCSAIL yields accurate absolute predictions of boat performance. Comparisons have not been made with actual
boat data. Predictions will depend on the sail force coefficients that are employed, and other elements of the hydrodynamic equations. This study does not provide a means of directly evaluating the accuracy of such inputs as "windage" drag, added resistance and the equivalent draft for centerboard boats. However, it is believed that these results do indicate that PCSAIL can be useful in the initial design stage, offering guidance for improvements before hull lines are developed. The ease with which sail force coefficients, and resistance equations can be altered, affords a means of readily determining the sensitivity of any design "improvements" to these inputs.

The empirical equations appear to yield reasonable estimates of the various hull parameters, based on the observed correlation coefficients. Such estimates appear to be a reasonable basis for establishing an initial design, however default values for displacement must be examined carefully to ensure that they are consistent with $C_p$ and hull dimensions. It is anticipated that experienced yacht designers will "adjust" such initial estimates, taking into consideration their particular design objectives and their own experience and data files.

PCSAIL is a simplified but effective computational device. It has been useful as a teaching aid because initial design guidance can be gained quite rapidly from input of only the basic hull and sail-dimensions on a home PC. Because the overall benefit to yacht designers is not known, PCSAIL can be made available to conference attendees on a limited basis, at no cost. The authors hope thereby to gain feedback from people actively involved in yacht design.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to the US Sailing Association, for providing copies of their comprehensive data on the North American IMS Fleet. They also wish to thank Messrs Andrew A. Magruder and Daniel DeWindt of US Sailing for their helpful cooperation. They also appreciate the help rendered by Messrs David J. Singer and Patrick J. Finn at the University of Michigan.

Special thanks are due to Lt. Commander Thomas M. Percy of the Canadian Navy for suggesting the use of the Excel Solver.

REFERENCES


APPENDIX

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Equations or Values for Hull Parameters</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement, lbs</td>
<td>DSPo = 0.0008( (LBT)^2 ) + 13.317(LBT) + 926.78 ( R^2 = 0.94 )</td>
<td>US Sailing Assoc. N.A. IMS Fleet * see below</td>
</tr>
<tr>
<td>Center of Buoyancy from forward perp.</td>
<td>( L_{mkp} = L_w/2 + 0.034 \frac{L_w}{L_{mkp}} ) for optimum, ( L_{mk} = 0.034 \frac{F_T}{F_L} = 0.35 ) ( L_m = % \text{ of } L_{mk} \text{ forward of } L_{mk} )</td>
<td>Larsson/Eliasson, 1994 p. 81</td>
</tr>
<tr>
<td>Center of floatation % of ( L_{mk} )</td>
<td>(1) deadrise hull ( L_d = 0.6144 \frac{L_w}{L_{mk}} + 1.9317 ) ( R^2 = 0.99 )</td>
<td>Data from Keuning and Sonnenberg, 1999</td>
</tr>
<tr>
<td></td>
<td>(2) &quot;U&quot; bottom hull ( L_d = 0.6305 \frac{L_w}{L_{mk}} + 4.5189 ) ( R^2 = 0.94 )</td>
<td></td>
</tr>
<tr>
<td>Initial righting moment, ft-lbs per degree @ 2 deg.</td>
<td>( RM_2 = 0.02181L^2 ) ( R^2 = 0.93 )</td>
<td>US Sailing Assoc. N.A. IMS Fleet * see below</td>
</tr>
<tr>
<td>Waterplane Area ( WPA, 8^2 )</td>
<td>( WPA = B_L L_{mk} \left[ 1.313 C_p + 0.0371 \left( L_w/V_L^{1.39} \right) - 0.0857 C_p \left( L_w/V_L^{1.39} \right) \right] )</td>
<td>Gerritsma et al., 1993</td>
</tr>
<tr>
<td>Wetted area canoe hull, ( S_w, 8^2 )</td>
<td>( S_w = \left[ 1.97 + 0.171 B_w/T_s \right] \left[ 0.65/C_n^{0.57} \right] ) ( V_L ), AWL( 8^{13} ) ( C_n = \text{midship section coeff.} )</td>
<td>Gerritsma et al., 1993</td>
</tr>
<tr>
<td>Prismatic Coeff.</td>
<td>( C_p = 0.56 ) &quot;optimum&quot; value at ( F_T = 0.35 )</td>
<td>Larsson/Eliasson, 1994, p. 79</td>
</tr>
<tr>
<td>Keel thickness ratio, ( t/c</td>
<td>\text{keel} )</td>
<td>( t/\text{keel root} = 0.15 )</td>
</tr>
<tr>
<td>Rudder thickness ratio, ( t/c</td>
<td>\text{rudder} )</td>
<td>( t/\text{rudder root} = 0.15 )</td>
</tr>
<tr>
<td>Centerboard thickness ratio ( t/c</td>
<td>\text{centerboard} )</td>
<td>( t/\text{centerboard} = 0.10 )</td>
</tr>
<tr>
<td>Daggerboard thickness ratio ( t/c</td>
<td>\text{daggerboard} )</td>
<td>( t/\text{daggerboard} = 0.10 )</td>
</tr>
<tr>
<td>Propeller pitch to Dia, P/D</td>
<td>( P/D = 0.7 )</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>Prop. Dev. Ratio</td>
<td>( A_x/A_x = 0.3 )</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>Propeller Drag Coefficient ( C_{prop} )</td>
<td>( C_{\text{prop}} ) fixed and locked = 1.2 ( C_{\text{prop}} ) fixed blades, free = 0.3 ( C_{\text{prop}} ) folding blades = 0.06</td>
<td>Larsson/Eliasson, 1994</td>
</tr>
<tr>
<td>Max. crewarm ( CRARM_{\text{crew}} )</td>
<td>Beam/2 = 1.5</td>
<td>Arbitrary</td>
</tr>
</tbody>
</table>

Table A-1

* US Sailing IMS Fleet was reduced to one of each class yielding a fleet of 836 boat designs. One 12 meter boat was dropped. \( B \) is the maximum beam, \( T \) is the hull depth, \( \approx T_d \), \( L \) is LSM0, measurement length, \( \text{(LSMO = L_{mk})} \). \( L.B.T \) is taken as \( L \) \( B^2 \) \( (T/B) \). \( B/T \) is the beam to depth ratio. DSP0 is the displacement in measurement trim. The displacement in sail trim is taken as: \( DSP_{\text{sail}} = 1.058 \) DSP0 + 752, pounds.
Hydrodynamic Equations

(Gerritsma et al. 11th CSYS)

\[ R_{tot93} = R_f + R_r + R_i + R_b \]

where

- \( R_f \) = frictional resistance on the hull and appendages.
- \( R_r \) = the residuary resistance of the canoe body.
- \( R_i \) = the induced resistance.
- \( R_b \) = the added resistance due to heel.

\[ R_f = q(S_{C_k} + S_{C_k} + S_{C_k}) \]

where

\[ q = 0.5 \rho_w V^2 \]
\[ \rho_w = 1.99 \text{ lb sec}^2/\text{ft}^4 \]
\[ S_{C_k}, S_{C_k}, S_{C_k} \] are the wet surfaces of the canoe body, keel and rudder.
\[ C_{C_k}, C_{C_k}, C_{C_k} \] are the related frictional coefficients therefore.

\[ C_f = 0.075/\log(R_e - 2)^2 \]
\[ R_e = \frac{V}{L/v} \]
\[ V = \text{boat velocity, ft/sec} \]
\[ L = 0.71 \text{ for canoe body} \]
\[ L = \text{avg. cord length for keel and rudder.} \]
\[ v = (1.28)10^{-5} \text{ ft}^2/\text{sec} \]

Additional frictional resistance for keel bulbs is calculated using the bulb area, and bulb length and the preceding equations. The resulting resistance, \( R_b \), is based on that value multiplied by the form factor suggested by Kenning and Binkhorst (1997).

\[ R_b = qS_{C_b}(1.5(d/l)^{1/2} + 7(d/l)^3) \]

where \( d/l \) is the max. ratio of diameter to length for the keel bulb.

\[ R_i = \Delta_i \left[ a_0 + a_1 C_p + a_2 L_{cb} + a_3 (B_w/T_e) \right] + \]
\[ a_4 (L_w/V_e^{1/3}) + a_5 C_p^2 \]
\[ a_6 C_p (L_w/V_e^{1/3}) + a_7 (L_{cb}^2) + a_8 (L_w/V_e^{1/3})^2 \]
\[ + a_9 (L_w/V_e^{1/3})^3 \] /1000

where each of the coefficients \( a_0-a_9 \) are given as a tabular function of the Froude number, \( F_n \), between 0.125 and 0.45 in Table A-VI.

For Froude numbers in excess of 0.45, (0.475-0.75) the residuary resistance is given by:

\[ R_i = \Delta_i \left[ c_0 + c_1 (L_w/B_w) + c_2 (A_w/V_e^{2/3}) + c_3 L_{cb} \right. \]
\[ + c_4 (L_w/B_w)^2 + c_5 (L_w/B_w)(A_w/V_e^{2/3}) \] /1000

Here the \( c's \) are also given as a tabular function of the Froude number, and also in Table A-VI.

In order that the residuary resistance be given as a continuous function of the boat speed (or \( F_n \)), it was necessary to write a table lookup function for values just above and below a given Froude number. Then the Residuary resistance was determined by linear interpolation between the two residuary resistance values for Froude numbers just above and below the given Froude number.

\[ R_i = \frac{F_{beel}^2/(q\pi T_e^2)}{A} \]

where

\[ F_{beel} = \text{sail force normal to mast.} \]
\[ T_e = \text{effective draft} \]
\[ T_e = A_1 T_e + A_2 (T_e)^2 + A_3 (B_w/T_e) \]

where

\[ A's \] are defined by:
\[ A_1 = 4.080 + 0.0370 \phi - 4.983 \phi^3 \]
\[ A_2 = -4.179 - 0.809 \phi + 9.9670 \phi^3 \]
\[ A_3 = 0.055 - 0.0339 \phi - 0.0522 \phi^3 \]

For center or dagger-board boats an equivalent draft is used, based on the empirical equation suggested by Kerwin, (MIT Report 78-11).

\[ T_{eq} = DHKA + ECMA \tan \left[ 0.5(EHMA/DHKA)^{1/2} + 0.2^*(EHMA/DHKA)^{3/4} \right] \]

where

\[ DHKA = \text{Draft of Fixed portion of keel} \]
\[ ECMA = \text{Extension of draft provided by the center or dagger-board} \]

\[ R_h = qS_{C_b} F_n^2 \phi \]

where

\[ \phi = \text{the heelf angle in radians.} \]
\[ C_b = \text{a heelf coefficient given by:} \]
\[ C_b = 6.747(TJT) + 2.517(B_w/T_e) + 3.710(B_w/T_e)(T_e/T) \]
\[ R_{prop} = 0.5 \rho V^2 C_{Dprop} A_p \]
\[ A_p = A_p/A_{0p} (1.067 - 0.229 P/D)^{xD^2/4} \]
\[ A_{0p}, C_{Dprop} \text{ see Table A-I} \]

\[ R_{waves}, \text{ from Claughton, (1999)} \]
\[ R_{waves} = 2 \rho g L f(V) 0.55 f(\beta) f(L_{40})[\text{hull and } F_n \text{ parameters}] \]

where

\[ L = L_{40} \]
\[ f(\beta) = \cos(\beta) \cos(40) \]
\[ f(V) = VR_{[1.08375(1.175exp-0.00248V_r^3)]} \]
\[ f(L_{40}) = 0.5059 \log(L_{40}) + 1 \]
\[ \text{[hull and } F_n \text{ parameters}] = 0.00146 \]
\[ + f(F_n) + f(K_{sp}) + f(L/B) + f(B/T) + f(LCB-F) \]
\[ f(F_n) = 0.00191(F_n - 0.325) \]
\[ f(K_{yy}) = 0.01575(Gyr - 0.25) \]
\[ f(L/B) = \frac{([5.23 \cdot L^2] - (5.23 \cdot 3.27^2))/8.494}{0} \]
\[ f(B_ff) = (0.000166(B_{ff}^0 - 4.443)) \]
\[ f((LCB - F) = 0.01 \left[ (LCB - LCF) - (-0.03) \right] + 0.0578 \left[ (LCC + LCF)^2 - (-0.03)^2 \right] \]

\[ K_{yy} = 0.222 \frac{(L_{oa} + L_{smh})}{2} \]
\[ Gyr = K_{yy}/L_{smh} - 0.03 + inc \]
\[ inc = \text{gyr} \text{adius increment, taken as zero} \]

**Selected Lift Coefficients from Claughton (1999)**

<table>
<thead>
<tr>
<th>Wind Angle</th>
<th>Main</th>
<th>Jib</th>
<th>Spinnaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>1.67</td>
<td>1.48</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>1.41</td>
<td>1.42</td>
<td>1.7</td>
</tr>
<tr>
<td>80</td>
<td>1.04</td>
<td>0.77</td>
<td>1.67</td>
</tr>
<tr>
<td>100</td>
<td>0.77</td>
<td>0.4</td>
<td>1.4</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Selected Parasitic Drag Coefficients Claughton (1999)**

<table>
<thead>
<tr>
<th>Wind Angle</th>
<th>Main</th>
<th>Jib</th>
<th>Spinnaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>0.02</td>
<td>0.02</td>
<td>0.1</td>
</tr>
<tr>
<td>50</td>
<td>0.08</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>80</td>
<td>0.25</td>
<td>0.57</td>
<td>0.8</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
<td>0.72</td>
<td>1.0</td>
</tr>
<tr>
<td>180</td>
<td>1.2</td>
<td>0.9</td>
<td>1.1</td>
</tr>
</tbody>
</table>

**Larsson/Eliasson Lift Coefficients**

<table>
<thead>
<tr>
<th>Apparent Wind Angle</th>
<th>Main</th>
<th>Jib</th>
<th>Spinnaker</th>
<th>Mizzen</th>
<th>Stay</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>1.5</td>
<td>1.5</td>
<td>0</td>
<td>1.3</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>1.5</td>
<td>0.5</td>
<td>1.5</td>
<td>1.4</td>
<td>0.75</td>
</tr>
<tr>
<td>80</td>
<td>0.95</td>
<td>0.3</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>100</td>
<td>0.85</td>
<td>0.0</td>
<td>0.85</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Larsson/Eliasson Viscous Drag Coefficients**

<table>
<thead>
<tr>
<th>Apparent Wind Angle</th>
<th>Main</th>
<th>Jib</th>
<th>Spinnaker</th>
<th>Mizzen</th>
<th>Stay</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>0.02</td>
<td>0.02</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>0.15</td>
<td>0.25</td>
<td>0.25</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>80</td>
<td>0.8</td>
<td>0.15</td>
<td>0.9</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>100</td>
<td>1.0</td>
<td>0.0</td>
<td>1.2</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>180</td>
<td>0.9</td>
<td>0.0</td>
<td>0.66</td>
<td>0.8</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Sail Force Equations**

**True Wind Strength**

All of the aerodynamic equations depend on the true wind velocity, \( V_1 \), which is considered to be zero close to the water's surface and vary with the height, \( z \), above the water in accordance with the Milgram's (1993) logarithmic equation:

\[ V_1(z) = V_1(10)[\ln(z) - \ln(z_0)]/\ln(10) - \ln(z_0) \]

where

\[ V_1(10) = \text{true wind velocity 10 meters (32.8 ft) above the water} \]
\[ z_0 = \text{the height at which the true wind is taken as zero} \]
\[ z_0 = \text{taken as 0.001 meters, or 0.003281 ft} \]

\[ V_1(z) = 0.1085736V_1(32.81)[\ln(z) + 5.719607] \text{ if } z \text{ is in feet.} \]

For sails \( CE \), \( z = (CE R_{ref} + F_{avg}) \cos \phi \)

\[ V_1(CE) = V_1(32.81)[0.621154 + 0.10857 \ln(CE R_{ref} + F_{avg}) + \ln(\cos(\phi))] \]

For windage drag on hull, \( z \) is taken as \( F_{avg}/2 \).

**Sail Driving and Heel Forces**

The main body of the paper has detailed how the sail Lift, \( L \) and Drag, \( D \) are computed from the sail areas and the sail force coefficients in terms of the effective apparent wind velocity \( U_a \) at the height of the center of effort of the sail combination. This apparent wind velocity and its direction are defined in the plane of the mast and the boat velocity in
accordance with the following equations, employed by Larsson and Eliasson (1994).

\[ U_a = \left( \left( V_0 \sin \beta \cos \phi \right)^2 + \left( V + V_0 \cos \beta \right)^2 \right)^{1/2} \]

where:
- \( \beta_t \) = the true wind direction relative to the boat’s course
- \( V \) = boat velocity along the course
- \( \phi \) = heel angle

\( \beta_a \) = the effective apparent wind angle relative to the boat course

This effective apparent wind angle is used to convert the computed lift and drag forces into \( F_{\text{drive}} \) and \( F_{\text{heel}} \) (eq. 13, 14), that are used in the equilibrium equations.

The heeling moment, due to \( F_{\text{heel}} \), is given by:

\[ M_h = F_{\text{heel}} (CE + F_{\text{avg}} + h) \]

where
- \( h \) = distance of center of lateral resistance to waterline
- \( h = T (0.414 - 0.165 T/T) \)

<table>
<thead>
<tr>
<th>Fringe No</th>
<th>( b_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_8 )</th>
<th>( a_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>-0.73054</td>
<td>36.3538</td>
<td>-0.008103</td>
<td>0.685234</td>
<td>1.069724</td>
<td>-36.8689</td>
<td>0.956947</td>
<td>-0.002171</td>
<td>1.272895</td>
</tr>
<tr>
<td>0.15</td>
<td>-0.36257</td>
<td>36.7179</td>
<td>-0.007583</td>
<td>0.253853</td>
<td>0.083723</td>
<td>-2.396533</td>
<td>0.365632</td>
<td>2.161639</td>
<td>0.000007</td>
</tr>
<tr>
<td>0.175</td>
<td>-1.20292</td>
<td>21.4464</td>
<td>-0.012701</td>
<td>0.087221</td>
<td>-2.446062</td>
<td>-31.9137</td>
<td>2.360706</td>
<td>0.000074</td>
<td>0.044345</td>
</tr>
<tr>
<td>0.2</td>
<td>11.19601</td>
<td>-1.51947</td>
<td>-0.017802</td>
<td>0.085170</td>
<td>-2.697508</td>
<td>-11.4168</td>
<td>0.856406</td>
<td>0.007024</td>
<td>-0.094934</td>
</tr>
<tr>
<td>0.225</td>
<td>22.51767</td>
<td>-16.1074</td>
<td>-0.006899</td>
<td>0.107029</td>
<td>-2.989448</td>
<td>7.187046</td>
<td>0.019261</td>
<td>0.037085</td>
<td>0.016271</td>
</tr>
<tr>
<td>0.25</td>
<td>26.56987</td>
<td>-74.9686</td>
<td>0.156621</td>
<td>0.165858</td>
<td>-0.858658</td>
<td>24.1217</td>
<td>10.4058</td>
<td>0.025348</td>
<td>0.056964</td>
</tr>
<tr>
<td>0.275</td>
<td>40.79556</td>
<td>-14.2565</td>
<td>-0.002123</td>
<td>0.028027</td>
<td>-0.313662</td>
<td>-33.0137</td>
<td>13.52177</td>
<td>0.035641</td>
<td>-0.015437</td>
</tr>
<tr>
<td>0.3</td>
<td>43.83759</td>
<td>-1.847964</td>
<td>0.367031</td>
<td>0.328437</td>
<td>3.671054</td>
<td>132.2966</td>
<td>66.8865</td>
<td>0.098660</td>
<td>-0.176219</td>
</tr>
<tr>
<td>0.325</td>
<td>89.29032</td>
<td>-360.0227</td>
<td>0.174592</td>
<td>0.490642</td>
<td>11.5437</td>
<td>311.1197</td>
<td>10.86156</td>
<td>0.014072</td>
<td>0.919303</td>
</tr>
<tr>
<td>0.35</td>
<td>213.8793</td>
<td>-801.7006</td>
<td>1.867877</td>
<td>0.538098</td>
<td>10.82979</td>
<td>697.6645</td>
<td>13.30615</td>
<td>0.006470</td>
<td>-5.626131</td>
</tr>
<tr>
<td>0.375</td>
<td>338.2354</td>
<td>-1085.134</td>
<td>1.864191</td>
<td>0.520729</td>
<td>-1.241751</td>
<td>831.1445</td>
<td>28.1823</td>
<td>0.234792</td>
<td>2.50047</td>
</tr>
<tr>
<td>0.4</td>
<td>586.5148</td>
<td>-1406.631</td>
<td>1.579749</td>
<td>0.267272</td>
<td>-2.944171</td>
<td>116.2891</td>
<td>51.4817</td>
<td>0.286845</td>
<td>0.179054</td>
</tr>
<tr>
<td>0.425</td>
<td>743.4107</td>
<td>-1706.283</td>
<td>2.436089</td>
<td>0.013553</td>
<td>-0.611618</td>
<td>857.4014</td>
<td>118.8065</td>
<td>0.366017</td>
<td>0.156667</td>
</tr>
<tr>
<td>0.45</td>
<td>1204.32</td>
<td>-3751.715</td>
<td>3.208577</td>
<td>0.356462</td>
<td>-132.0424</td>
<td>1488.269</td>
<td>165.3408</td>
<td>0.523220</td>
<td>1.376102</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fringe No</th>
<th>( b_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_8 )</th>
<th>( a_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.475</td>
<td>180.1004</td>
<td>-31.9037</td>
<td>-7.48141</td>
<td>2.396042</td>
<td>3.969023</td>
<td>0.004948</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>243.6904</td>
<td>-44.3551</td>
<td>-11.16555</td>
<td>2.719046</td>
<td>3.959703</td>
<td>0.009976</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.525</td>
<td>282.8783</td>
<td>-31.5993</td>
<td>-13.03721</td>
<td>2.79456</td>
<td>4.342965</td>
<td>0.01598</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>313.4106</td>
<td>-36.5027</td>
<td>-14.41978</td>
<td>2.926117</td>
<td>4.864427</td>
<td>0.021524</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.575</td>
<td>337.0534</td>
<td>-49.1029</td>
<td>-16.08957</td>
<td>2.419519</td>
<td>4.797029</td>
<td>0.026064</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td>364.8552</td>
<td>-52.3868</td>
<td>-10.00112</td>
<td>2.407099</td>
<td>5.076768</td>
<td>0.011454</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.625</td>
<td>324.7367</td>
<td>-51.3125</td>
<td>-15.43488</td>
<td>2.234148</td>
<td>3.655386</td>
<td>0.01999</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.65</td>
<td>301.3396</td>
<td>-50.3881</td>
<td>-15.32306</td>
<td>2.056875</td>
<td>3.564318</td>
<td>0.023582</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.675</td>
<td>292.8571</td>
<td>-45.8635</td>
<td>-15.59455</td>
<td>1.877925</td>
<td>1.992607</td>
<td>0.014014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>284.4641</td>
<td>-25.1456</td>
<td>-15.15423</td>
<td>1.703981</td>
<td>0.817912</td>
<td>0.014575</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-VI “a”, and “c” Coefficients