Analysis of Wave Making Resistance And Optimization of Canting Keel Bulbs

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ABSTRACT
In upwind sailing conditions, bulbs of canting keel sailboats operate close to the water surface and therefore induce non-negligible changes in the wave system, thus influencing the wave making resistance of a sailboat. The bulb could produce relatively positive as well as negative effects, and therefore its design is suitable for optimization.

A case study for a modern 100-foot canting keel sailboat is presented. The design space was explored to determine the bulb shape that would produce favorable interference with the hull wave system, reducing the total resistance of the sailboat for an upwind sailing condition, using the Response Surface Optimization (RSO) methodology developed by the authors. In the 1,000 bulb variations that were performed using an advanced parametric modeler, the volume and location of the center of gravity of the bulb were fixed so that the stability of the sailboat was not altered and, therefore, the computed total resistance became a direct measure of merit of the bulb design.

An outlook is given to a combined optimization of hull and bulb in order to gain optimum improvement.

NOTATION

\begin{align*}
\rho & \quad \text{Density of water} \quad [\text{kg/m}^3] \\
F_R & \quad \text{Froude number} \quad [-] \\
g & \quad \text{Acceleration due to Gravity} \quad [\text{m/s}^2] \\
\quad \quad \quad & \quad \text{LCB} \quad \text{Longitudinal Center of Buoyancy} \quad [\text{m}] \\
\quad \quad \quad & \quad \text{LPP} \quad \text{Length between perpendiculars} \quad [\text{m}] \\
\quad \quad \quad & \quad \text{RE} \quad \text{Reynolds number} \quad [-] \\
\quad \quad \quad & \quad \text{RT} \quad \text{Total resistance} \quad [\text{N}] \\
\quad \quad \quad & \quad \text{RV} \quad \text{Viscous resistance} \quad [\text{N}] \\
\quad \quad \quad & \quad \text{RW} \quad \text{Wave making resistance} \quad [\text{N}] \\
\end{align*}

INTRODUCTION
The development and use of bulbous bows on ships has demonstrated that it is possible to interfere positively with the ship’s generated wave system and reduce the wave making resistance for a given design condition. While the canting keel concept improves the upwind sailing performance of a sailboat by adding stability, it generates a secondary negative effect on the wave making resistance due to the close proximity of the bulb to the free surface. With increased heel and cant angle the bulb of a canting keel sailboat approaches the free surface and the pressure distribution due to its displacement induces free surface waves. These waves interfere with the wave system generated by the hull and induce changes in the wave making resistance of the sailboat. Figure 1 shows, for a modern 100-foot canting keel sailboat, the variation of wave making, viscous and total resistances due to cant angle for an upwind sailing condition at 20 degrees heel, 1 degree yaw and \(F_R = 0.38\). A decrease of more than 3.5% in wave making and 1% of total resistance can be observed with decreasing cant angle.
The authors’ objective was to determine if there is, similarly to bulbous bows on ships, a potential for optimizing the shape of canting keel bulbs by minimizing the negative effect of the bulb on resistance when in close proximity to the free surface. A modern 100-foot canting keel sailboat design was used in the case study presented.

In order to perform a complete and efficient survey of the design space topology, the case study investigation was performed using the RSO methodology developed by Fassardi and Hochkirch (2006). Developed to explore in a systematic manner the relevant features of a given design for a given set of design parameters, and corresponding bulb geometry, that would produce the least total resistance for the given sailing condition.

THE RSO METHODOLOGY

The RSO methodology is described and demonstrated for a hull development case study by Fassardi and Hochkirch (2006). However, a general description is included here.

The basic assumption in the RSO methodology is that a measure of merit, such as the total resistance, is as a function of the parameters that define the design space. Designers often try to understand how this function varies by evaluating the changes in the measure of merit that result from subjective changes in the design parameters and, based on this evaluation, settle on the perceived “best” combination of these. Investigations are performed by trial-and-error or systematically following a direction of improvement as the investigation is iterated. The simulation and experimentation work necessary to derive measures of merit with these approaches is typically very intensive, resulting in a relatively small number of design variations and a poor investigation of the design space.

The RSO methodology also hinges on the feasibility of representing this function mathematically. Once a function can be represented mathematically, means for evaluating it become available. Furthermore, the function’s maximum (or minimum) can be computed using an optimization solver allowing for the search of the “best” design in an objective manner.

Given the multiple dimensionalities of these functions (i.e. as many dimensions as design parameters chosen for the design space), these functions are often referred to as “surfaces” or “response surfaces” due to the input-output relationship they represent. The process of finding the maximum (or minimum) of a function is called optimization, and when the function is represented by a response surface the procedure is referred to as Response Surface Optimization (RSO).

A solution to the problem of design optimization could be achieved if we could produce an interpolating multi-dimensional response surface of the measure of merit such that it would retain the topological features of the design space using relatively few design variations. The “best” measure of merit, and corresponding design parameters could then be obtained from the response surface using an optimization solver. The RSO methodology provides this solution, since it features the three elements that are needed: a) a sampling strategy that allows to retain the design space topology with relatively few design variations uniformly distributed over the design space, b) an algorithm that can interpolate the resulting data and create the response surface, and c) an optimization solver capable of finding the global maximum (or minimum).

In order to provide a random, uniform sampling of parameters within a given design space a Sobol sequence is used. Being actually a quasi-random strategy, the Sobol sequence ensures to always have a well represented design.

![Figure 1. Variation of resistance with cant angle.](image-url)

The resulting total resistances and corresponding bulb design parameters were fitted using radial basis functions to provide: a) a continuous mathematical representation of the data (i.e. a response surface) and b) means to allow for the evaluation of the total resistance anywhere within the design space.

Finally, an optimization solver was used in conjunction with the response surface to find the set of design parameters, and corresponding bulb geometry, that would produce the least total resistance for the given sailing condition.
space, in a statistical sense, with increasingly finer resolution as more samples are produced, while avoiding clustering the design parameters. An additional advantage of this strategy is that the algorithm avoids “grid lines” and thus, gives a more stable basis for subsequent multidimensional interpolation, much more effectively than other strategies such as the Monte Carlo.

The proposed algorithm to generate the response surface is based on radial basis functions or RBFs. The main strength of RBFs is their ability to elegantly and accurately interpolate scattered data in arbitrary dimensions, without using data on a grid. Several other algorithms could be used such as polynomials, splines, neural networks, but RBFs have proven to be computationally fast and accurate.

CASE STUDY

Baseline Design

A modern 100-foot canting keel sailboat was used as baseline design. Main particulars of this sailboat are summarized in Table 1 and geometry is depicted in Figure 2. For simplicity and in order to concentrate on the wave making effects due to the bulb variations the supporting bulb strut, daggerboard and rudders were not included.

<table>
<thead>
<tr>
<th>Hull</th>
<th></th>
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<tbody>
<tr>
<td>LPP</td>
<td>27.56 m</td>
<td></td>
</tr>
<tr>
<td>Displacement</td>
<td>33.11 tonnes</td>
<td></td>
</tr>
<tr>
<td>Beam at WL</td>
<td>4.48 m</td>
<td></td>
</tr>
<tr>
<td>Draft Canoe Body</td>
<td>0.59 m</td>
<td></td>
</tr>
<tr>
<td>Draft</td>
<td>4.90 m</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bulb</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>4.10 m</td>
<td></td>
</tr>
<tr>
<td>Width</td>
<td>0.82 m</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>0.62 m</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Main particulars of the baseline design.

Parametric Variations

One thousand bulb designs were generated using FRIENDSHIP-Modeler (2004), a software system for the advanced parametric modeling of sailboats and ships. Within this system, the bulb geometry is directly generated from a small set of form parameters, while the characteristics and constraints of the bulb are maintained and the shape is optimized with respect to fairness. The modeling approach is based on multiple nested optimizations of uniform B-spline curves defining sectional properties of the bulb for each longitudinal position. These curves are built from a flexible selection of properties. Finally, the FRIENDSHIP-Modeler can be easily coupled to a sophisticated Sobol sequence generator to sample the design space as defined by design parameter ranges.

In order to simplify the study for the demonstration purposes of this paper, and because only effects on resistance were investigated, design variations that produced changes in stability were not considered. This permitted to use the total resistance as a direct measure of merit. Therefore, two constraints were specified such that the volume and center of gravity of all bulb variations were the same as for the baseline bulb. The parametric model of the bulb allowed control of the top and bottom center plane curves of the bulb and it directly employed the sectional area curve in order to specify the volume distribution.

Figure 3 shows the completely parametrically modeled baseline bulb. Of about 40 form parameters that could be used to characterize the bulb, seven were selected and varied for the case study, while the rest were kept constant or adjusted by the modeler to maintain maximum fairness of the shape. In a real design project, it would be necessary to define a larger design space and select more varying parameters. The parameters selected for variation are illustrated in Figure 4. In a first step, the top and bottom center plane curves were modeled by a set of parameters including tangency and curvature information. In a second step the sections of the bulb were created where the generation process implicitly handles the constraint on sectional area as taken from the sectional area curve and maintains maximum fairness of the surface. For more details see Harries (2001), Hochkirch (2002), FRIENDSHIP (2004).
Figure 4. Main parameters of the bulb model.

As the longitudinal center of buoyancy (LCB) of the bulb was free to move within some bounds relative to the bulb length the global position of the bulb was then shifted to match the overall LCB of the baseline configuration. The length of the bulb was allowed to vary rather freely between 3 and 8 meters and the center plane curves of the bottom and top sections were free to change to adjust the extent and location of the maximum elevation. In a first study the beaver tail width was free to vary within a reasonable range but kept fixed for subsequent studies as it was felt that its influence is not accurately reflected by the numerical method. Only the selected design parameters were allowed to vary during the parametric variations, allowing the FRIENDSHIP-Modeler to optimize the remaining ones with respect to fairness of the sections.

Table 2. Free variables and ranges of bulb parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>BulbCPCLength</td>
<td>3.0m</td>
<td>8.0m</td>
<td>4.098m</td>
</tr>
<tr>
<td>BulbCPCDraft</td>
<td>0.1m</td>
<td>0.4m</td>
<td>0.295m</td>
</tr>
<tr>
<td>BulbCPCxMid*</td>
<td>30 %</td>
<td>60 %</td>
<td>46.3 %</td>
</tr>
<tr>
<td>BulbTopHeight</td>
<td>0.2m</td>
<td>0.4m</td>
<td>0.33m</td>
</tr>
<tr>
<td>BulbTopMidPos*</td>
<td>30 %</td>
<td>60 %</td>
<td>46.1 %</td>
</tr>
<tr>
<td>Xcb*</td>
<td>45 %</td>
<td>50 %</td>
<td>48.5 %</td>
</tr>
</tbody>
</table>

The design space was sampled using a Sobol sequence which, in a semi-stochastic manner, distributed the 1,000 bulb design variations within the ranges listed in Table 2.

Measure ofMerit

Since the authors’ objective was to investigate the potential for optimizing the shape of canting keel bulbs to minimize the negative effect of the bulb on resistance when in close proximity to the free surface, the effects of stability were isolated by fixing the volume and center of gravity of the bulbs. This allowed the use of total resistance, $R_F$, produced by each design variation as a direct measure of its merit.

The total resistance, $R_F$, was computed by the nonlinear free surface potential flow code FRIENDSHIP-Flow as the sum of the wave making resistance, $R_W$, and the viscous resistance, $R_V$. FRIENDSHIP-Flow uses a Rankine source distribution on the hull and the free surface and iterates to satisfy the dynamic and kinematic boundary conditions on the elevated free surface. Dynamic sinkage and trim of the hull are simultaneously adjusted within these iterations.

The wave making resistance $R_W$ was computed by pressure integration over the hull panels. Alternatively, the computed wave heights were analyzed according to Eggers et al. (1967) and the wave pattern resistance is computed from the decomposition of the spectral wave energy. This method is generally known to be less sensitive to discretization errors.

Since the potential flow method does not include viscous resistance components these are approximated within FRIENDSHIP-Flow by using the ITTC ship correlation line,

$$C_F = \frac{0.075}{\log_{10}(R_F - 2)^2}$$

Considering the local velocities on the submerged hull panels, the total viscous resistance can be computed as the sum of the viscous resistance on each panel as:

$$R_V \approx \frac{\rho}{2} C_F \sum_{i} v_i^2 A_i$$

where $\rho$ is the water density, $N$ is the number of panels, $v_i$ is the flow velocity on the panel and $A_i$ is the area of the panel. Even though this is a very basic approximation it has proven to be useful for preliminary studies as more sophisticated methods have significant demands on computational efforts and are therefore prohibitive for the explorations to be performed here.

The computational model was further simplified to include only the bare hull and the bulb. Bulb strut, daggerboards and rudders were not included in order to keep the computations simple and fast. While the resistance of the bulb strut may be of negligible magnitude, the resistance of the daggerboards and rudders may not and therefore, the predicted total resistance is missing a significant component. However, as the study was focused on the principal wave making components, those due to the omitted lifting surfaces are not expected to alter the findings presented here. The resistance ratios shown, however, should be thought of being indicative only.

As a further simplification two typical upwind sailing conditions were selected for this study. For both, heel and leeway were fixed at 20 and 1 degrees respectively, cant angle 40 degrees and boat velocities were $V = 12$ knots ($F_R = 0.38$) and 13.4 knots ($F_R = 0.42$). This simplification is based on the assumption that the different bulb designs will not induce differences in lift as a function of leeway and therefore the drag polars for the different bulb designs will be parallel to each other with no cross-overs. Table 3
Response Surface

Several algorithms are commonly used to construct response surfaces. Among the most popular are polynomials, splines, neural networks and radial basis functions. For this case study, the response surface was derived by means of radial basis functions (RBFs) of the Gaussian type. This technique was selected because of its computational efficiency, ease of use, and fundamentally, because they work well with multi-dimensional scattered data. The application of RBFs on scattered data was performed initially by Hardy (1971).

Optimization

The optimization solver used in this study was a Generalized Reduced Gradient solver which, like other gradient-based solvers, is meant to find local solutions. Response surfaces, like the one derived in this study, will feature irregularities that most likely would lead the solver to find local solutions instead of the desired global solution. Traditional ways of dealing with this problem include starting the solver from several initial conditions derived from the knowledge of the response surface topology, or by a trial-and-error method by which the solver is started from a multitude of points (multistart) to see which point led to the “best” (global) of the local solutions. The former approach is very difficult since prior knowledge of the response surface topology is often unavailable or difficult to determine. The latter approach could be time consuming or, if not organized and conducted in a systematic manner, would lead to the same local solution identified several times, thereby leading to an inefficient global search.

The optimization solver used in this study included a “multi level single linkage” clustering technique by which initial conditions are randomly sampled and clustered into groups that would likely lead to the same local solution. The solver is run next to find the local solution within each cluster followed by a Bayesian analysis to determine when to stop sampling new clusters. The solution obtained with this method converges with a high degree of probability to the global solution searched.

SENSITIVITY STUDIES

Two questions that were asked were: What could be the magnitude of the interference effects that could be produced by a) changing the bulb location and b) making it very long. To answer the first question, a sensitivity study was performed where the bulb was moved longitudinally by a significant amount. Certainly, this is not a valid alternative as the sailboat would assume unrealistic trim angles. However, the resulting effects would give a qualitative idea of what could be gain if the bulb is located at an atypical location.

Figure 6 shows results of the computations done for the baseline sailboat sailing in the selected upwind condition at 12 knots ($F_R = 0.38$) fixed in trim. Even at this lower $F_R$ there is a significant effect of interference with the hull’s wave system which could amount to a change of more than 5% in wave making resistance if the bulb is shifted forward or backwards by more than 5 meters, a shift that is certainly not within the feasible range.

It is interesting to see is that the viscous resistance is also somewhat reduced for the forward shifted positions. This is due to the reduced wetted surface area in the forward part of hull that results from the local wave height that is also reduced due to the wave interference effects.

This sensitivity study suggests that while positioning the bulb in the middle of the hull may be the optimum to achieve longitudinal hydrostatic balance, it does not seem to be the optimal location from the standpoint of...
hydrodynamic resistance. A twin bulb configuration comes immediately into mind which might be superior in terms of calm water resistance.

To answer the second question, results from the 1,000 bulb variations were used to examine the effects of bulb length on the trade-off between wave making resistance and viscous resistance. As bulb length increases wave making resistance is expected to decrease, while the viscous resistance will increase due to the increase in wetted surface. Only for the purpose of assessing the magnitude of change, the bulb length was changed within the unrealistic range of 3 to 8 meters. Results gave a qualitative idea of the effect of length and wetted surface changes for a canting keel bulb in the selected upwind sailing condition.

Figure 7 shows resistance versus bulb length for the baseline design sailing in the selected upwind condition at 12 knots (F_R = 0.38). There is a clear trend that wave making resistance can be considerably reduced when using a longer bulb. However, as the wetted surface increases with the longer bulb the viscous resistance increases at a much higher rate favoring shorter bulbs. Although the other parameters used to characterize the bulb also had an effect on resistance, none of them showed an effect as strong as the length.

The RSO methodology intends to minimize this problem by offering the possibility of identifying regions of the design space where the “best” design may exist from a limited number of samples of the measure of merit. For the case study presented in this paper, the “best” bulb design could be determined simply by sorting the total resistance, and corresponding design parameters, of the 1,000 variations generated. However, one of the objectives of this study was to generate, thanks to the several assumptions made, a large number of variations which measures of merit could be used to verify the RSO methodology. This verification was performed in four steps. It was important to verify that:

1. the “best” set of design parameters that were obtained from the 1,000 variations did indeed produce a design better than the baseline;
2. the total resistance corresponding to RSO-derived design parameters were in agreement with those derived using the CFD code for the same set of parameters;
3. the total resistance corresponding to the “best” RSO-derived design parameters using only 30 design variations was in agreement with the total resistance of the “best” design identified from the 1,000 variations; and
4. the sets of “best” design parameters clustered in a region of the design space, permitting the identification of the region of optimum performance.

Table 4 shows the total resistance for the baseline bulb configuration and those corresponding to the “best” set of design parameters found using the different methodologies for F_R = 0.42. “Baseline” is the parent bulb from which the 1,000 design variations were generated. Best_1000 and Best_30 are the bulb design in the 1,000 and the first 30 bulb variations, respectively, that produced the least total resistance computed by the CFD panel code. RSO_1000 and RSO_30 are the bulb designs found by the RSO methodology to produce the least total resistance when using 1,000 or 30 design variations to generate the response surface, respectively. Results indicated as “RSO” correspond to the Response Surface Optimization prediction, while those indicated as “CFD” correspond to the total resistance computed by CFD for the design parameters found by RSO. The RSO_30 bulb is shown in Figure 8.

For the set of design constraints selected, the optimization shifted BulbTopMidHeight forward while BulbCPCDraft was shifted to a relative backward position. It is noted that the viscous resistance approximation may not be sufficiently accurate for the relatively steep buttocks in the aft of this bulb.
Table 4. Total resistance for baseline and “best” bulbs.

<table>
<thead>
<tr>
<th>Design</th>
<th>Total Resistance [N]</th>
<th>Better Than Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>7,653</td>
<td></td>
</tr>
<tr>
<td>Best_1000</td>
<td>7,598</td>
<td>-0.72%</td>
</tr>
<tr>
<td>RSO_1000</td>
<td>CFD 7,613</td>
<td>-0.52%</td>
</tr>
<tr>
<td></td>
<td>RSO 7,575</td>
<td>-1.02%</td>
</tr>
<tr>
<td>Best_30</td>
<td>7,809</td>
<td>-0.58%</td>
</tr>
<tr>
<td>RSO_30</td>
<td>CFD 7,603</td>
<td>-0.65%</td>
</tr>
<tr>
<td></td>
<td>RSO 7,348</td>
<td>-3.99%</td>
</tr>
</tbody>
</table>

Table 4 shows that, within the framework of the CFD code used, a better design than the baseline could be achieved. The RSO methodology under-predicts the total resistance but is able to identify regions of the design space where the “best” design may exist. Notably, the RSO methodology was able to find a design of comparable performance as the best one found within the 1,000 parametric variations when using a response surface derived from only 30 designs. This constitutes a very attractive capability of the methodology, since it would be expected to provide objective guidance in development programs where, due to the inherent limitations of resources and time of sophisticated technologies such as RANS CFD and physical model tests, relatively few samples of measures of merit can be derived.

Figure 9 through Figure 15 show various sections of the design space, the relative locations of the baseline design, and the parameters of the best designs obtained with the parametric variations and RSO methodology. Figure 9 shows a clear benefit in having a relatively short bulb with a relatively deep bottom section. Perhaps as an indication of a multi-modal (several optima) response surface, Figure 9 shows a benefit in having a relatively shallow top section, but also relatively deep one, as predicted by the RSO methodology. Figure 10 indicates that it would be beneficial to locate the center of volume at a location forward that of the baseline. However, according to the RSO prediction the Xcb could be located further aft. Figure 11 shows the benefit of locating the maximum draft of the bottom section between 50% to 60% of the bulb length from the nose, while Figure 12 suggests a more
forward position for the maximum height of the top section of the bulb. Figure 13 suggests that a bulb with a top section relatively flatter than the bottom section would be beneficial. However, a 1:1 relationship may work as well, as indicated by the RSO prediction. Figure 14 shows the advantage of having shorter and wider bulbs, relative to the baseline while Figure 15 shows that a shorter bulb does not necessarily result in lesser wetted surface except when correctly combined with other design parameters.

**DETERMINISTIC SEARCH**

In order to explore a possible additional reduction of resistance a deterministic search strategy was performed using directly the measure of merit as computed by the CFD method, which is possible under the simplifications discussed here but may be prohibitive when using more sophisticated numerical methods. A Tangent Search method, as described by Hilleary (1966), was started from the bulb design with the least resistance in the 1,000 design variations performed so far (Best_1000). This algorithm consists of exploratory moves along the variable axes followed by global moves in the decent search direction found by the successful exploratory moves. If a constraint bound is approached, a tangent move in hyperspace is conducted tangential to the constraint either to keep the search in the feasible domain or to bring it back into the feasible domain, see Hilleary (1966) for more details. Figure 16 shows the optimization history for this search where an additional resistance reduction of 0.05% could be achieved, indicating that the already gained performance improvement could not be substantially more improved. For more complex systems (i.e. more free variables), however, it is the author’s experience that this technique is very efficient in detecting further performance advantages.

The optimization shifted the maximum height of the bulb as much forward as allowed by the design space constraints while the position of maximum draft was shifted to an aftmost position. It should be noted that the viscous resistance approximation may not be sufficiently accurate at the relatively steep buttocks in the aft of this bulb. Also the constraint set considered in this study, again for simplicity, did not limit the maximum depth of the complete configuration which, however, is a strong demand in most real design projects.

**CONCLUSIONS**

The methodology presented allowed for a quick and efficient exploration and evaluation of the design space of a canting keel bulb close to the free surface for an upwind sailing condition. Through the case study presented it was possible to validate the capability of the RSO methodology to identify, from a limited sample of designs and corresponding measures of merit, a bulb design that may perform optimally from the standpoint of resistance.

Results from the sensitivity studies that were...
performed showed that the bulb and hull wave systems can interfere positively with each other resulting in beneficial reductions in resistance. The parametric variation of the major bulb design parameters showed that bulb length and fore-aft location were critical parameters. However, the extreme alternatives that showed significant advantages may not be of use in real world applications. Nevertheless, they show a trend that may be worth exploring with atypical configurations. On this regard, as shown by Fassardi and Hochkirch (2006), parametric variations of the hull can be performed, which combined with simultaneous variations of the bulb designs, can increase the flexibility of the design space and result in attractive benefits.

Long bulbs can considerably reduce wave making resistance. However, due to their increased wetted surface the viscous penalty may not pay off and shorter bulbs may be preferred. It must be noted that especially for short blunt bulbs the computational method used to estimate the viscous resistance may not be the best choice and more sophisticated methods such as RANS CFD should be used for final verification.

The results presented in this paper are limited to the specific design and sailing conditions of the case study. For designs and/or sailing conditions where the ratio of wave making to total resistance increases (i.e. at higher Froude numbers) the effects discussed here may become more relevant and the optimization of the bulb design from the standpoint of wave making resistance may constitute an important objective in the design program.

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REFERENCES


