The Third International Conference on Innovation in High Performance Sailing Yachts, Lorient, France

ADVANCED STRUCTURAL ANALYSIS METHOD FOR AEROELASTIC SIMULATIONS OF SAILS

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This paper presents an advanced method to predict the structural behaviour of modern fiber-membrane sails and its validation by on-board sail photographic survey. The presented structural analysis method is an improvement of a direct stiffness method that shows good numerical stability and is able to treat nonlinearities with the same level of performance of a dynamic method, but less time consuming. In order to achieve this performance, a damping-like force has been added to the structural system. By tuning a damping factor, the behaviour of the structural analysis code can be switched from a classical static method to a dynamic-like one. Thus, this method allows running accurate analyses of fiber-membrane sails with battens by taking into account both the geometric non-linearity and wrinkling behaviour of membrane structures in a timely manner. Furthermore, it is also very effective when sails are coupled with rigging elements, e.g. when the luff sag calculation is required. This advanced structural analysis method is coupled with a nonlinear vortex lattice method to enable a proper aeroelastic simulation of sails in upwind conditions, within the SMAR-Azure technology. The SMAR-Azure fully integrated aeroelastic analysis method has been extensively validated using on-board photographic survey. In this paper, the comparison between the calculated and the real flying sail shapes of the fiber-membrane sail plan of the 55ft race boat “Living Doll” is presented.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>C</td>
<td>Stiffness tensor (N.m$^{-2}$)</td>
</tr>
<tr>
<td>D</td>
<td>Damping factor (N)</td>
</tr>
<tr>
<td>$F_D$</td>
<td>Damping loads (N)</td>
</tr>
<tr>
<td>$F_E$</td>
<td>External loads (N)</td>
</tr>
<tr>
<td>$F_I$</td>
<td>Internal reaction loads (N)</td>
</tr>
<tr>
<td>I</td>
<td>Identity matrix ( )</td>
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<tr>
<td>$K_E$</td>
<td>Elastic stiffness matrix (N.m$^{-1}$)</td>
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<tr>
<td>$K_G$</td>
<td>Geometric stiffness matrix (N.m$^{-1}$)</td>
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<td>$\delta$</td>
<td>Displacement (m)</td>
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<td>Principal Strain 1 ( )</td>
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<td>$\varepsilon_W$</td>
<td>Wrinkling strain correction vector ( )</td>
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<tr>
<td>$\sigma_2$</td>
<td>Principal stress 2 (N.m$^{-2}$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Wrinkling direction (rad)</td>
</tr>
<tr>
<td>N</td>
<td>Total Number of degrees of freedom</td>
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<tr>
<td>NR</td>
<td>Newton-Raphson</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-Structure Interaction</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Model</td>
</tr>
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<td>LOA</td>
<td>Length Over All (m)</td>
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<tr>
<td>LWL</td>
<td>Waterline Length (m)</td>
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<td>AWA</td>
<td>Apparent Wind Angle (deg)</td>
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<td>True Wind Angle (deg)</td>
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<td>True Wind Speed (knots)</td>
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<tr>
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<td>Boat Speed (knots)</td>
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1 INTRODUCTION

This paper presents an advanced structural analysis method used within the aeroelastic analysis tool developed by SMAR-Azure Ltd for the simulation of the structural behaviour of fibre-membrane sails. The structural simulation of sails is one of the challenging problems of the current marine engineering due to its strong nonlinearities. The aim of the work shown in this paper is the development of an accurate solution method that could be easily used in a timely manner in the everyday work of the sail designers. The paper describes the innovative theoretical approach to solve the finite element analysis for membrane sails and its validation via on-board photographic survey. The full fluid-structure interaction (FSI) solution turns out to be very robust and accurate. The code is able to solve even large sail-plan FSI problems in a reasonable time and is able to deliver the full optimization processes of sails on a simple Windows based personal computer.

Specifically, Chapter 2 describes some of the main enhancements included in the SMAR-Azure structural code. The addition of a damping factor to the structural system increases the robustness of the code itself. The implementation of a robust wrinkling model increased the accuracy of the results. Furthermore, the possibility to include rig elements into the analysis make the SMAR-Azure analytical code able to take into account important aspects of the sail-rig system, like the influence of the forestay tension on the flying sail shape of headsails.
Chapter 3 describes the experimental testing campaign carried out in conjunction with Doyle Sails New Zealand in order to validate and improve the code.

2 STRUCTURAL ANALYSIS METHOD

2.1 OVERVIEW

The structural analysis code developed by SMAR-Azure Ltd to evaluate the flying sail shape of fiber-membrane sails is a nonlinear finite element method. In particular, a direct stiffness method is used to solve the structural problem [1][2][3]. Since a nonlinear problem is solved, the Newton-Raphson’s method is used to find the deformed equilibrium state of the structure. Geometric and wrinkling nonlinearities are taken into account. The stress-strain relation of the sailcloth material is considered as linear instead. Considering one step of the Newton-Raphson’s (NR) algorithm, the structural system is linear and its behaviour is described by the equation:

\[ (K_E + K_G) \delta = \Delta F = F_E - F_I \]  

(1)

The global stiffness matrices \( K_E \) and \( K_G \) are assembled adding the contribution of every finite element. CST (Constant Strain Triangular) membrane elements are used to model the sailcloth. The sailcloth can be modelled as made of an isotropic, orthotropic or anisotropic material. In case the real fiber layout has to be taken into account, a stacking procedure (classical lamination theory) is used to compute the anisotropic stiffness matrix of the laminate. Battens can also be included as beam elements. The wrinkling behaviour of the sailcloth is taken into account by a dedicated model that avoids compression stresses. The entire structural analysis can be coupled with a nonlinear vortex lattice method in order to obtain a proper fluid-structure interaction (FSI) simulation.

2.2 DAMPING-LIKE FACTOR

One of the main issues of using a direct stiffness method with sail structures is its small robustness when the stiffness of the system becomes negligible. Indeed, the geometric stiffness \( K_G \) is null at the beginning of the analysis because the sailcloth internal stresses are null. The elastic stiffness \( K_E \) could have some zero-stiffness points as well because the out-of-plane stiffness of a membrane is negligible. For these reasons, the system of equations (1) could easily become singular and the solution diverges. In order to avoid diverging solution and improve the robustness of the code, a damping-like force has been added to the external loads. If we consider a step of the NR algorithm as a time-step, a damping force can be considered proportional to the incremental displacement \( \delta \) of that particular step. Thus, for each FEM node, a damping force would be:

\[ F_D = D \delta \]  

(2)

where \( D \) is a damping factor. Considering the \( i^{th} \) row of the system of equations (1) and adding the damping force we obtain:

\[ \sum_{j=1}^{N} (K_{ij} \delta_j) = \Delta F_i + D \delta_i \]  

(3)

Extending the concept to the whole system, the equation (1) becomes:

\[ (K_E + K_G + D I) \delta = \Delta F \]  

(4)

The addition of the damping diagonal matrix avoids the global stiffness matrix to become singular. Moreover, tuning the damping factor maintains the analysis stable. Indeed, increasing the damping factor \( D \) makes the analysis more robust but increases the number of Newton-Raphson iteration needed to get the equilibrium of the system. For that reason, it has been implemented an automatic adaptation of the damping factor during the analysis in order to get the best compromise between numerical stability and computational time. In Figure 1, the effect of the damping on the Newton-Raphson algorithm is explained. In that graph (Fig. 1), the slope of the tangent to the thick curve is proportional to the stiffness of the analysed structure. If that slope is almost horizontal, the system tends to be singular and the displacement would be unreasonably large. Adding the damping force increases that slope, avoiding the singularity of the linear system.

![Figure 1: Newton-Raphson algorithm with (red lines) and without (black lines) damping](image)
In addition to the increased robustness of the code, this technique is also very effective when strong nonlinearities, like wrinkles, has to be taken into account.

2.3 WRINKLING MODEL

Since the stress-strain relation of a membrane element is linear, its stiffness under tension or compression is the same. This is not a realistic assumption because the sailcloth, having a negligible bending stiffness, cannot resist to a compressive load and it wrinkles. In order to predict that behaviour, a wrinkling model has been implemented [4]. At the end of each NR step, the strain state is evaluated for every membrane element and the related stress state is computed from the stress-strain relationship:

\[ \sigma = C \varepsilon \]  

(5)

At that stage, a wrinkling state is evaluated according to a mixed stress-strain criterion:

- Taut: \( \sigma_2 > 0 \)
- Slack: \( \varepsilon_1 \leq 0 \)
- Wrinkled: \( \varepsilon_1 > 0 \) and \( \sigma_2 \leq 0 \)

If a slack or wrinkled state is detected, a strain correction is computed and applied to the membrane element. Under wrinkling condition, the strain correction values satisfy the following relationship:

\[
\begin{bmatrix}
\varepsilon_1 \\
0 \\
0
\end{bmatrix}^\varphi = C^\varphi \left( \varepsilon_1^\varphi + \varepsilon_w^\varphi \right)
\]  

(6)

Indeed, when the sailcloth is wrinkled, only the stress along the wrinkling direction \( \varphi \) is positive (\( \sigma_1 \)). All the other components of the stress tensor have to be null. In order to make the convergence of the analysis easier, a relaxation factor is applied to the wrinkling strain correction. The wrinkling model works well with isotropic, orthotropic or anisotropic materials. In case of non-isotropic materials, equation (6) is solved upon the numerical computation of the wrinkling direction.

2.4 RIG COUPLING – LUFF SAG

Within the SMAR-Azure technology, the sail structure can also be coupled with the rig structures. An example of the coupled analysis features is the computation of the luff sag for headsails, which forestay bending has to be computed. When running rig-sail coupled analyses, the structural equilibrium for sail and rig are solved independently within each iteration. Two independent systems of equations are built and solved. From the sail analysis results, the reaction forces at the connection points with the rig are applied on the rig finite element model. Once the rig analysis is performed, the displacements at the connection points are applied to the sail structure as imposed displacement constraint and the next NR iteration starts. A flowchart of the sail-rig coupling is shown in Figure 2.

![Flowchart of Sail-Rig coupling process](image)

Specifically, when computing the foresails luff, the forestay is modelled as a series of cable elements pinned at the ends. The internal tension of the forestay is considered constant during the whole analysis. The value of the sailing forestay tension is an input of the analysis. As a result, the converged forestay sag is not dependent on the mechanical properties of the forestay.

3 VALIDATION

The SMAR-Azure has carried out a broad testing campaign in order to validate the structural and aeroelastic analysis code implemented for the solution of the sail-membrane structure as well as for the rig-sail structure. Specifically, the validation campaign has involved numerical simulations as well as experimental tests on-board. The present paper describes a specific experimental test carried out in collaboration with Mr. Richard Bouzaid, head designer of Doyle Sails New Zealand. The following sections describe the results of experimental tests carried out on a sailplan designed and optimised by Mr. Bouzaid using the SMAR-Azure technology for the boat Farr 55ft ‘Living Doll’ (Fig. 3). The specifics of the boat Farr 55ft ‘Living Doll’ are shown below.

- LOA = 16.76 m
- LWL = 15.78 m
- Beam = 4.57 m
- Draft = 3.50 m
- Displacement = 8980 kg

It is important to say that one of the major problems of the testing campaign has been the retrieving of the flying sail shapes from the picture in known sailing conditions.
3.1 ON-BOARD DATA

The on-board photographic survey of the sail set Main+Jib2 (MJ2) is reported in this paragraph. Table 1 reports the wind condition of the sea trials. Table 2 reports the sails trimming conditions used in that wind condition.

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<td>BS [Knots]</td>
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<tr>
<td>HEEL [deg]</td>
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<tr>
<td>LEEWAY [deg]</td>
<td>4</td>
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Table 1: Wind data

<table>
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<tr>
<td>Main [deg]</td>
<td>1</td>
</tr>
<tr>
<td>Jib [deg]</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table 2: Sails sheeting angles

Figure 4: Flying shape of the Mainsail

Figure 5: Flying shape of the Jib

3.2 NUMERICAL SOLUTIONS

A full aeroelastic analysis of the sail plan and relative forestay interaction has been run. The sail plan FEM was built by 5143 membrane elements (3045 for the mainsail and 2098 for the jib), which was taking into account the real fiber layout, scrim and fill, as designed by Mr. Bouzaid.

In order to carry the aerodynamic analysis out, wind data recorded on-board are used to define the boundary conditions (as from table 1). A logarithmic profile of the true wind speed is used to take into account the atmospheric boundary layer and the twist of the apparent wind direction. Sails were trimmed as in the sea trials (as from table 2), by moving the clew in the correct position. Some of the aeroelastic analysis results are described heretofore. In order to get the numerical solution of the flying sail shape, 8 aerodynamic and structural analysis iterations (FSI) were carried out. The resulting maximum displacement, which means the maximum difference between design and flying sail-shape was 8.2cm for the mainsail and 15.4 cm for the jib.

Figure 6 shows the aerodynamic pressure and the flying sail shapes compared with the design shape. From those two pictures, it is possible to note that the aeroelastic simulation is able to get a typical effect of the mainsail-jib aerodynamic interaction: the jib makes the pressure on the luff of the mainsail to become negative (defined by the blue region in the pressure plot) and the cloth of the mainsail moves windward (defined by the green area in the flying sail shape plot).
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Figure 6: Aerodynamic pressure (top) and deformed structural mesh (bottom) after the full aeroelastic analysis (or FSI)

Figure 7 and 8 show the principal stresses and strains on respectively the mainsail and the jib. The structural analysis takes into account the real fiber layout and finishing material of the sails thanks to the anisotropic formulation of the membrane elements. Thanks to the wrinkling model, the thermal plot of the principal stress 2 shows no negative stress in the sailcloth. Some wrinkled elements are shown in Figure 9, where direction and density of the lines plotted in each triangular element identify respectively the wrinkling direction and its amount.

Figure 7: Aeroelastic analysis results of the Mainsail. From left to right: principal stress 1 [Pa], principal stress 2 [Pa], principal strain 1 [%]

Figure 8: Aeroelastic analysis results of the Jib2. From left to right: principal stress 1 [Pa], principal stress 2 [Pa], principal strain 1 [%]

Figure 9: Wrinkled elements
The structural analysis of the jib has been coupled with the forestay, tensioned as measured during the sea trials. The resulting sag is shown in Figure 10.

![Figure 10: Forestay sag](image)

### 3.3 COMPARISON

The experimental validation test was carried out by comparing the flying sail-shape as evaluated by the SMAR-Azure aeroelastic code (FSI) with the one measured during the sea trials (see figure 4 and 5). Specifically, the main geometric data of the sail sections (camber, draft, twist) were extracted by analysing the draft stripes on the pictures taken during the sea trials. In order to compare the numerical flying sail shape with the one measured when sailing, it was necessary to extrapolate the same data from the numerical flying sail-shape (FEM mesh). Figure 11 shows the draft stripes on the numerical mesh and the draft stripes on the real sail shape.

![Figure 11: Mainsail qualitative comparison.](image)

It is important to say that discrepancies between the two measurements are expected for a number of reasons:

- the computed flying sail shape is a discrete model formed by small flat triangular elements, although a fine granularity is used;
- for the jib, the real luff sag is unknown
- the top section shapes are difficult to evaluate from the pictures, because of the smaller sail sections (they are actually flat)

Figure 12 shows the comparison among the designed mainsail, the numerically calculated one and the real flying sail shape. From top to bottom, the camber, draft and twist measured at the draft stripes are plotted. Figure 13 shows similar comparisons for the jib 2. Considering the mainsail, it is possible to note that the SMAR-Azure code is able to evaluate the sail structural behaviour in a very accurate way. Indeed, both the numerical and the real flying sail shape indicate that the mainsail shape, when compared with the designed one, tends to be fuller (camber increases), the draft moves backward and the leech opens slightly in the middle and closes at the top (as shown from the twist graph). Considering the numerical and real mainsail flying shape, it is possible to note that camber and twist distribution along the sail vertical profile is accurately evaluated, while the draft shows a slightly higher discrepancy, which it is believed due to the difficulty to evaluate it using a graphical approach. As expected, the higher discrepancies are at the head section, because of the difficulty to measure it from the pictures.

![Figure 12: Flying sail shape comparison of the Mainsail: from top to bottom, camber, draft and twist at the draft stripes.](image)
Considering the Jib2, it is possible to note that both the numerical and the real flying sail shape indicate that the J2 shape, when compared with the designed one, tends to be fuller (camber increases), the draft moves backward and the leech opens. Looking at the flying sail-shape picture it is possible to note that the jib luff moves backwards (sags) causing an increase of the camber in the forward area.

Considering the numerical and real mainsail flying shape, the results are very positive. Unfortunately, as the luff sag was not measured for the Jib2, it is difficult to appreciate the reasons for the higher discrepancy on the draft evaluation on it. Indeed, the twist and camber evaluated are almost identical.

Figure 13: Flying sail shape comparison of the Jib 2: from top to bottom, camber, draft and twist at the draft stripes.

4 CONCLUSIONS

A fully aeroelastic analysis tool (or FSI) for fiber-membrane sails has been presented. The emphasis has been placed on the enhancement of the structural analysis code and its validation via on-board photographic survey. The increased robustness of the code due to the damping factor (section 2.2), the implementation of a robust wrinkling model (section 2.3) and the possibility to run sail-rig coupled analysis (section 2.4) made the SMAR Azure technology to become an effective and accurate analysis tool for sailmakers and yacht designers. The extensive work of enhancement and validation carried out in conjunction with Doyle New Zealand led to excellent results. Further developments will concern the extension of the sail-rig coupling to the entire sailplan. Indeed, the whole rig could be included in the aeroelastic analysis and coupled with mainsail and headsails. It would allow taking into account the stiffness of the rig and the influence of the tuning of shrouds and stays.

5 ACKNOWLEDGEMENTS

Special thanks to the technical team of SMAR-Azure Ltd, Dr. Donald W. MacVicar and Mr. Stephen Jordan, for their continuous and pro-active contribution to the development of the method and graphics presented in this paper.

And thank you to Mr Michael Hyatt, owner of the ‘Living Doll’.

6 REFERENCES


7 AUTHORS BIOGRAPHY

Dr. Sabrina Malpede is the co-founder and Managing Director of SMAR Azure Ltd. Since 1997, she has been involved in developing a scientific approach for sail-design, firstly during her doctorate and then within SMAR Azure Ltd for the development of products and services required by the Industry. She is a graduate with honours in Aeronautical Engineering at the University of Naples (Italy), has a Ph.D. in Sail Design from the University of Glasgow (UK).

Fabio D’Angeli currently holds the position of Research and Development Engineer at SMAR-Azure Ltd. He is involved in the development of the aerodynamic and structural analysis methods of the SMAR-Azure Ltd technology. He graduated with honours in Nautical Engineering at the University of Genoa (Italy).
Richard Bouzaid is a director of Doyle sails NZ, manufacturers of Stratis membranes for the Doyle group of sailmakers. He has been involved with many significant sailing programs; he has been a sail trimmer on board Alinghi winning the Americas Cup in Auckland in 2003, and sail trimmer and coordinator onboard Whitbread Race’s winning Yamaha. Richard designs for many international race projects including Hugo Boss, Leopard and Team Korea.